Influence of Family Background on College Attendance: Evidence from Tuition Policy Changes in Russia

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Abstract

Current study analyzes the choice of families for their children’s tertiary education and the influence of family background and existing tuition policies on these choices. We explore the natural experiment of changes in tuition policies happened during 1990-2006 years in the Russian Federation (passage from state-subsidized education to mixed forms of tertiary education: on a state-subsidized and full-tuition basis). We construct and estimate the model to analyze the different influences of family background and family income on college attendance within different educational systems. First, we allow family income (together with family background) to affect students’ abilities at the moment of college entrance. Second, we allow for heterogeneity in the quality of college education. Third, we introduce an admission selection performed by colleges, which is strictly determined by applicants’ abilities and varies depending on the educational type (state-subsidized or full-tuition) or college quality (lower and higher quality institutions). The results suggest, that educational reforms in Russia during 1990-2006 years have significantly increased the importance of family income in determining college attendance, especially on a full-tuition basis. Moreover, family income, compared to parents’ educational background, determines to a larger extent the sorting of students among colleges with different quality of education.

Key words: educational choice, family background, tuition policies

JEL: J24, I20, I28

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1 Introduction

During the Soviet period, state universities were the only higher education institutions in the Russian Federation. Government completely financed the tertiary education. Nevertheless, the number of available places in universities, colleges and vocational education institutions was limited. Potential students obtained their admission on a competitive basis. Admission tests selected high-ability candidates for study, so students’ choices for school were restricted by their ability levels.

The return to tertiary education has been increasing substantially after market liberation. This in turn led to a significant increase in the demand for tertiary education and, therefore, to an increase in the number of private universities and creation of non-budget studying options (on a full-tuition basis) in the state universities. Thus, in 2006 the percentage of full-tuition students reached 59%, including 43% studying in state universities and 16% studying in private universities (Gohberg et al. (2007)).

Today individual educational choices are limited not only by personal abilities but also by families’ capacities to finance education. In the both types of tertiary education admission tests are mandatory, but the required level of knowledge is lower for full-tuition posts. Thus, families’ financial capacities to pay for tertiary education weaken the minimum ability requirements. Nevertheless, the educational process is the same for state-subsidized and full-tuition students, as well as final certificates of degrees obtained. Therefore, the existing educational system provides two possibilities for Russian youth: either they should be smart enough to enter the universities, or their parents could ”buy” the university entrance for them.

In the current study, the main interest is to analyze the choice of families for their children’s tertiary education and the influence of expected labor-market outcomes and existing tuition policies on these choices. Another aim of this study is to test whether the return to ”full-tuition” education is expected to be the same as for ”free” education, or it is lower / higher for students with lower abilities (who actually paid for tertiary education attainment). In other words, whether the tertiary education is seen as a signal on the labor market or does the personal productivity play a major role in wages formation.

We construct and estimate a model describing household decisions. Following previous researchers the estimation are conducted in the form of sequential choices. Firstly, households make a choice between three alternatives after secondary school graduation: working, continue studying and obtaining the first level of tertiary education, continue studying and obtaining the second level of tertiary education. At this stage the households’ decisions are influenced by the expected future earnings as well as expected probabilities to find a job in general and to find a job corresponding to the education obtained. Secondly, households make a decision about financing the tertiary education (conditionally on the
possibility to enter universities on the state-subsidized basis), in other words a decision to acquire additional level of education which is unavailable on the state-subsidized basis due to low personal abilities. Estimation is conducted taking into account the unobserved heterogeneity of candidates. We suppose that higher-abilities students tend to choose the state-subsidized education, and lower-abilities students need to pay tuition fees completely. So dividing all the young population entering the tertiary education system into these two groups allows us to use the information about their educational abilities and to analyze the demand function for tertiary education and factors determining it for higher-abilities and lower-abilities youth. The data for this analysis are taken from the Russian Longitudinal Monitoring Survey and Russian Survey of Education for period 1995-2006.

The paper is organized as follows. Section 2 presents an Institutional context and a brief description of the Russian market of tertiary education. Section 3 discusses literature and contributions made by the current study. Section 4 describes the theoretical model. Section 5 discusses the model and introduces the methodology of econometric analysis. Section 6 presents results and Section 7 concludes.
2 Institutional Context

Figure 1 presents the ratio of students admitted to the 1st level of tertiary education establishments according to the category of their studies: studies within public educational institutions (on budget and non-budget basis (with paying in full tuition fees)) and studies within private educational institutions (thus non-budget basis).

Figure 1: 1st Level Tertiary Education Admission, by years. In thousands of students and in % to the total admission rate.

Source: Education in the Russian Federation (Gohberg et al. (2007))

Figure 2 presents the ratio of students admitted to the 2nd level of tertiary education establishments (higher education) according to the category of their studies: studies within public educational institutions (on budget and non-budget (with paying in full tuition fees) basis) and studies within private educational institutions (thus non-budget basis (with paying in full tuition fees)).

One could observe the substantial increase in the share of non-budget places within public educational establishments as well as an increasing number of private educational establishments. The increase in the total admission rate is mainly driven by the growing
Figure 2: 2nd Level Tertiary Education Admission, by years. In thousands of students and in % to the total admission rate.

Source: Education in the Russian Federation (Gohberg et al., 2007)

number of available non-budget places (with full tuition fees).
3 Literature Overview

The theoretical and empirical framework of this study is represented by three main directions: analysis of educational choices and future labor market outcomes, analysis of family background and educational choices, and analysis of tuition policies and their influences on educational attainment.

The first set of papers analyzes the educational attainment and influence of future expected labor market outcomes (such as possibility of finding a job and expected earnings) on these choices. The first theoretical works analyzed choice between time devoted to schooling and work (Levhari and Weiss (1974)) as well as the investments into education as a risky marketable asset (Williams (1978), Williams (1979)). They showed that the increase in the expected net rates of human capital depreciation and in the expected rates of return to employment is positively associated with an optimal level of education. At the same time, the increase in local absolute risk aversion, in the variance of the returns to education and in the variance of future wages decreases significantly the optimal level of educational investments.

The pioneer empirical work on this subject was presented by Willis and Rosen (1979), where they allow the demand for college education to depend on expected future earnings. Later following the work by Keane and Wolpin (1997) several econometricians have estimated structural dynamic models of schooling and working decisions. The detailed review of the schooling attendance and market outcomes estimation within a structural framework is presented in Belzil (2006). Differences in the initial endowments (personal abilities) and in the expected market outcomes explain different schooling attainment by population cohorts, for example for white and black youth (Keane and Wolpin (1997), Keane and Wolpin (2000)). These studies analyze the educational choices as the length of study (attending higher school, than college).

Similar approach has been used to analyze attendance of different types of educational institutions - mainly the analysis is conducted on the choice of majors and the influence of expected earnings on this choice. Recently, Arcidiacono (2004) has considered sequential models of college attendance and major choices. He shows that even if there are significant differences in payments for majors on the labor markets, future earnings explain very little of the ability sorting across majors.

There is large evidence about positive intergenerational correlation in educational attainment (in other words, children of more educated parents obtain higher educational levels). However, previous researchers argue that parents' education has rather indirect influence on schooling attainment: via hereditarily effect on the personal children abilities (genetic endowment) and preferences towards higher educational levels (cultural endowment); via family income and thus higher investments in children education, their skills, health, learning and motivation (see Haveman and Barbara (1995)) for corresponding papers overview]. Children who grow up in a poor low-income family tend to have lower
educational and labor market achievements than children from more affluent families. Keane and Wolpin (2001) focus on the analysis of the importance of listed above channels in determining the intergenerational correlation in educational attainment. Using a structural model approach they model the influence of parental money transfers on the education attainment and find that: first, more educated parents do indeed make larger transfers to their children especially during college studies; and, second, that these transfers are responsible for higher completed educational levels observed for the children of more educated parents. Post-estimation counterfactual simulations show that the equalizing of parental transfers (making them independent of parental schooling) significantly reduces the completed schooling levels of children whose parents are college graduates, but nevertheless does not significantly increase the attendance levels of children of less educated parents.

The current paper also makes a contribution to the question of importance of financial channel in the intergenerational correlation in educational levels by analyzing household investments in children education, which they have to make in order to obtain higher educational levels than their children’s abilities allow (taking into account the competitive environment during educational institutions entrance).

The third set of paper that makes up the framework of the current analysis is dedicated to the analysis of tuition influences on schooling attainment and simulation of tuition policies. Previous works on the effects of tuition on the schooling attainment show that an increase in tuition and a decrease in financial aids lead to declines in enrolment; at the same time enrolments are more sensitive to grant awards than to loans or work study, especially for students from low income families, minorities and non-residents (see Heller (1997) for the review of correspondent studies).

Recent studies provide evidence on the relationship between tuition policies and educational attainment from the general equilibrium model of student choices and universities decisions. Thus Epple and Romano (2002), Epple et al. (2003), Epple et al. (2006) provide an extensive analysis of the allocation of students by income and ability among colleges in the USA as well as an evidence of tuition fees charged by different level colleges to students that differ in abilities. The findings of their works suggest that there is a strong hierarchy of colleges by the educational quality provided to students, and consequently this hierarchy is characterized by income and ability stratification. They show that the highly ranked schools (with correspondingly higher educational quality) exercise a substantial variation of tuition fees with income together with discounts to more able students. Lower ranked schools charge also lower tuition to more able students. There could be two explanations of such college policies. Firstly, it could be an evidence of the importance of peer effects in educational achievements, so that students with higher ability levels pay lower tuition in equilibrium because of the positive externality they have on other students through the peer group effect. Secondly, such strategies underline the importance of presence of high-ability students for colleges as a signal of correspondent educational quality that would increase its prestige. Authors propose also the policy that would increase the financial
aid targeting toward lower income students and thus encouraging poor students to attend higher quality colleges.

Nevertheless, there is no a lot of evidence on the joint influence of tuition policies, family background and wages expectations on the educational choices. Below we list three of such works that combine and analyze these aspects of educational choices in a joint form and which are used as the main basis for the current research.

Arcidiacono (2005) has recently estimated a structural model combining the analysis of individual educational choices effecting by future earnings and tuition policies of the US colleges. More precisely he estimates how individuals decide where to submit applications and, conditional on being accepted and offered financial aid, in which college to enrol and what major to study. At the same time the school decisions about acceptance and financial aids are estimated. By this model he explains the different attendance and effect on earnings for white and black students. He shows that race-based advantages (advantages in admission and financial aid) have little effect on earnings but do have a significant effect on black students enrolment at top-tier schools and college attendance. His approach consists in structural estimation of a four stage sequential model: at the first stage individuals choose where to submit applications; at the second stage schools make admissions and financial aid decisions; at the third stage individuals decide which school to attend, conditional on previous school decisions; and, finally, at the fourth stage individuals enter the labor market with correspondent wages.

Magnac and Thesmar (2002) estimate a dynamic schooling model analyzing the increase in schooling attainment observed between 1980 and 1993 in France. They looked at three factors that potentially could be behind this increase: an increase in the return to education, a decrease in the direct and physics costs of schooling and a decrease in academic requirements (an increase in the success probability given enrolment). They show that the observed increase in attainment is most likely explained by a decrease in academic selectivity.

Boudarbat and Montmarquette (2007) analyze educational attainment of Canadian youth paying a special attention to the parents’ education and taking in the account the probabilities that students will be able to find an employment related to their field of study when evaluating lifetime earnings after graduation. They suppose a myopic form of expectations proposed by Manski (1993), according to which students are assumed to form their wage expectations by observing the earnings of comparable individuals who are currently working. They show that lifetime earnings have no statistically significant impact when the parent of the same gender as the student has a university education.

In the current study we follow the Arcidiacono (2004), Arcidiacono (2005) and the approach of Boudarbat and Montmarquette (2007) concerning the way how students form their expectations. There is a large number of works dedicated to the analysis of the validity of the rational expectations assumption in the context of educational choices (Manski:1993). We use a myopic form of expectations mainly because of the structure of the
data available for our analysis.

Additionally to the previous literature, we take into account the options of obtaining state-subsidized education or paying in full tuition fees. First, we concentrate on the analysis of factors influencing a decision to acquire additional level of education on the paid basis. Especially we focus on the question which family background stimulate the additional “buying” of education even if the skill level is not enough to enter correspondent educational level and how the expectations of labor market outcomes influence these choices. Second, we focus on the difference of expectations among households paying and not for education. We also take into account the expectations of employment and expectations of having a correspondent level of occupation while taking a decision about educational attainment.
4 Model

This section describes the model, which analyzes the importance of personal abilities, family background and family income in educational choices depending on the structure of the tertiary education system. We distinguish five types of tertiary education systems:

- Homogeneous quality of college education; all college education is state-subsidized;
- Homogeneous quality of college education; college education is state-subsidized or on a full-tuition basis;
- Heterogeneous quality of college education; all college education is state-subsidized;
- Heterogeneous quality of college education; all college education is on a full-tuition basis;
- Heterogeneous quality of college education; college education is state-subsidized or on a full-tuition basis.

The model allows us to analyze the different influences of family background and family income on college attendance within these structures of the educational system.

The current model is an extension of the model of educational choice presented in Belley and Lochner (2007). Belley and Lochner (2007) analyze the increase in the importance of family income for the educational achievement in the USA from the early 1980s to the early 2000s. In their 2-period model, the family income enters as the initial individual resource and as a variable that determines college tuition (which is assumed to be higher for families with greater financial resources because of the financial-aid policies for students with lower-income backgrounds - a typical practice in US colleges).

Here we extend their model in the following ways. First, we allow family income (together with family background) to affect students’ abilities at the moment of college entrance. Second, we allow for heterogeneity in the quality of college education. Third, we introduce an admission selection performed by colleges, which is strictly determined by applicants’ abilities and varies depending on the educational type (state-subsidized or full-tuition) or college quality (lower and higher quality institutions). We restrict the model in that individuals do not have a capacity to borrow, however, the current model can be easily generalized to the case with borrowing constraints.

The results suggest, that within educational systems, which provide tertiary education through both types of colleges on a state-subsidized and on a full-tuition basis, family income plays a significant role in determining college attendance on a full-tuition basis. Moreover, family income, compared to parents’ educational background, determines to a larger extent the sorting of students among colleges with different quality of education.
4.1 General Framework of the Model.

We assume that $N$ individuals live $T+1$ periods. In the first period (which corresponds to the first year after secondary school graduation), they choose whether to receive a higher education degree (college education) or not. Instead of college education, one can enter the labor market directly. Study at college is possible only during the first period and covers one period of life, and then all individuals enter labor market and work for $T$ periods.

$\theta_i$ represents individual abilities at the beginning of the first period. We assume here that $\theta_i$ is a function of family background ($Ed_{p,i}$ - parents’ education is used as a proxy for family background), family income ($Inc_{p,i}$), and individual ability endowment ($\xi_i$):

$$\theta_i = \theta\left(Ed_{p,i}, Inc_{p,i}, \xi_i\right)$$

We assume that individual abilities $\theta_i$ increase according to parents’ education and parents’ income. Such representation of individual abilities at the moment of college entrance takes into account three channels of intergenerational transmission of abilities: genetic transmission of abilities from parents; different non-monetary investments (time, efforts) in children’s education by parents with different educational levels; and different monetary investments in children’s education that occurs before secondary school graduation (due to the different households’ income levels).

Let $W_i$ stay for initial family resources available to spend on education of an individual $i$ ($W_i \geq 0$). $W_i$ is a part of a yearly family income $W_i = k_i \cdot Inc_{p,i}$, where $0 \leq k_i < 1$ (thus, a family cannot spend all its income on children’s education, but can also decide not to spend anything).

$\beta$ stands for future discount rate ($0 < \beta \leq 1$).

$\nu(\theta_i)$ represents the consumption value of college attendance.

An individual $i$ makes educational decision maximizing his/her net lifetime value function (which we specify below).

Colleges make their admission decisions maximizing the level of abilities of their students. Admission decisions by colleges are restricted by the number of available places in colleges (we assume that this number is given exogenously; however, assuming the endogenous choice of the number of student places will not change significantly our results).

At the beginning we assume that all colleges provide the same level of education (homogeneity in college quality), and that college education is fully state-subsidized. In the following subsections, we relax these assumptions and analyze the changes in the roles of family background and family income in college attendance.
4.2 The case of homogeneous quality of college education on a state-subsidized basis.

Variable $s_i$ describes the individual choice: it is set to 1 if an individual $i$ decides to enter college and 0 otherwise.

$w_s(\theta_i)$ reflects earnings per period in the labor market of an individual $i$ with educational level $s$. We assume that both education and individual abilities influence wages positively. Thus, $w_1(\bar{\theta}) > w_0(\bar{\theta}) > 0$ for all $\bar{\theta}$, $w_s(\theta_1) > w_s(\theta_2)$ for all $\theta_1 > \theta_2$. Individual utility function $U(x)$, where $x$ is the consumption in a correspondent period (in other words the wage), is an increasing, strictly concave function.

$t$ represents direct costs of college (living expenditures, transport, etc.) and does not include any tuition fees.

Therefore, we can write the lifetime value function for non-college attendance of an individual $i$ as following:

$$V_{0,i} = U(W_i) + U(w_0(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta}$$  (2)

The lifetime value function for college attendance of an individual $i$ is given as:

$$V_{1,i} = U(W_i - t) + \nu(\theta_i) + [U(w_1(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} - U(w_1(\theta_i))]$$  (3)

Thus, the net lifetime gain from college could be written as:

$$V(W_i, \theta_i) = [U(w_1(\theta_i)) - U(w_0(\theta_i))] \cdot \frac{1 - \beta^T}{1 - \beta} - U(w_1(\theta_i))$$

$$+ \nu(\theta_i) + U(W_i - t) - U(W_i)$$  (4)

Influence of $\theta_i$ and $W_i$ on the net lifetime gain from college can be expressed as follows:

$$\frac{\partial V(W_i, \theta_i)}{\partial \theta_i} = [U'_{w_1} \cdot w'_1(\theta_i) - U'_{w_0} \cdot w'_0(\theta_i)] \cdot \frac{1 - \beta^T}{1 - \beta} - U'_{w_1} \cdot w'_1(\theta_i) + \nu'(\theta_i)$$  (5)

$$\frac{\partial V(W_i, \theta_i)}{\partial W_i} = U'_{W_i}(W_i - t) - U'_{W_i}(W_i)$$  (6)

Because of the utility function concavity, the partial derivative of the net lifetime gain from college with respect to initial available resource for education is positive. The positive sign of the partial derivative also comes from the fact that with the increase in family income (and, thus, in available resources for education) the direct costs of education count for a smaller part in the family income, and this fact results in the lower loss in utility of the first period due to college attendance. The patterns of the influence of individual abilities on the net lifetime gain from college are determined by the financial returns to college in terms of the increase in wages conditional on abilities and by the difference in schooling
tastes among individuals with different abilities. If the increase in abilities \( \theta_i \) does not yield a higher increase in wages after college graduation, compared to the increase in wages with a secondary education degree (in other words, if \( w'_1(\theta_i) \leq w'_0(\theta_i) \)), the net lifetime gain from college could be a decreasing function with respect to abilities (but still could have a positive relationship with abilities if the consumption value of schooling is strongly positively affected by abilities and has a convex form, thus \( \nu'(\theta_i) > 0 \) and \( \nu''(\theta_i) > 0 \)). If the increase in abilities \( \theta_i \) yields a higher increase in wages with a college degree compared to potential wages with secondary education (in other words if \( w'_1(\theta_i) > w'_0(\theta_i) \)), so that \( U_{w_1} \cdot w'_1(\theta_i) - U_{w_0} \cdot w'_0(\theta_i) > 0 \), the net lifetime gain from college will increase with abilities. The last assumption is usually imposed in educational choice models and consistent with empirical data (see Belley and Lochner (2007) for more detailed analysis). Therefore, the net lifetime gain from college is an increasing function with respect to abilities and initial financial resources available for education. And, thus, there exists the level of ability \( \tilde{\theta} \) such that for all \( \theta_i > \tilde{\theta} \) the net lifetime gain from college is positive, and for all individuals with the level of ability \( \theta_i < \tilde{\theta} \) the net lifetime gain from college is negative. Consequently, individuals with ability level \( \theta_i > \tilde{\theta} \) will prefer to attend a college, and individuals with ability level \( \theta_i < \tilde{\theta} \) will not apply for college admission.

Moreover, \( \tilde{\theta} \) for individuals with higher initial resources will be lower. \( U(W_i - t) - U(W_i) \), being negative, is decreasing in absolute value with the increase in initial resources, and that is why the net lifetime gain from college \( V(W_i, \theta_i) \) becomes positive for lower levels of abilities \( \theta_i \).

As we assume in this part of the model, government completely finances the college education. Nevertheless, the number of available places in colleges is limited. Potential students obtain their admission on a competitive basis. Admission tests select high-ability candidates for study, so students’ choices for school are restricted by their ability levels. \( I_{p1}(\theta_i) \) represents this restriction: the value of this variable is set to 1 if an individual \( i \) who decided to enter a college will be admitted, and 0 if individual \( i \) will not be admitted. Even if individual \( i \) chooses to go to college, but his \( \theta_i < \theta_{min} \), he will not be able to attend. \( \theta_{min} \) is the minimal required level of ability to be admitted. Therefore, for all \( \theta_i \geq \theta_{min} \implies I_{p1}(\theta_i) = 1 \), and for all \( \theta_i < \theta_{min} \implies I_{p1}(\theta_i) = 0 \). Where \( \theta_{min} \) is determined on the basis of the number of applications to colleges and the number of available places in colleges \( M: \sum_{i=1}^{N} (I_{p1}(\theta_i)) = M \).

Thus, individual \( i \) solves the following maximization problem, choosing \( s_i \):

\[
V_i = \max_{s_i} \left\{ (1 - s_i) \cdot V_{0,i} ; \ s_i \cdot \left[ V_{1,i} \cdot I_{p1}(\theta_i) + V_{0,i} \cdot (1 - I_{p1}(\theta_i)) \right] \right\}
\]

subject to:

\[
s_i \cdot t \leq W_i \quad (7)
\]

After receiving students’ applications colleges solve the following maximization problem, determining cut-off ability level \( \theta_{min} \) and their admission decision \( I_{p1}(\theta_i) \) for each student \( i \)
(note that $I_{p1}(\theta_i) = 0$ for those who are not applying to college, thus, for those with $s_i = 0$):

$$I_1 = \max_{I_{p1}(\theta_i)} \left\{ \sum_{i=1}^{N} (I_{p1}(\theta_i) \cdot \theta_i \cdot s_i) \right\}$$

subject to:

$$\sum_{i=1}^{N} (I_{p1}(\theta_i)) = M$$

(8)

Figure 3 illustrates the solution of students’ and colleges’ maximization problems: equations 7 and 8.

Figure 3: Homogeneous college education on a state-subsidized basis

Note: ....

For the above figure, the shaded region $(\theta abcde)$ corresponds to the distribution of characteristics $\theta_i$ and $W_i$ in the population of secondary school graduates. Because of the positive influence of family income on current abilities and initial resources, the last two characteristics ($\theta_i$ and $W_i$) are positively correlated. The area $gfbcdeh$ shows individuals with a positive net lifetime gain from college (those who are applying for college admission). The region filled with diagonal stripes $(kbcj)$ illustrates individuals gaining college admission (according to the selection rule $I_{p1}$ applied by colleges). In the presented figure, the proportion of students with a positive net lifetime gain from college (thus, the propor-
tion of students willing to attend college) is higher than the number of available places in colleges. This fact totally corresponds to the empirical evidence for the Russian tertiary education system: the number of students applying to colleges is 1.3-7.2 times higher than the number of admitted students (differences by regional levels in the proportion of the number of admitted students to the number of students applying to state-subsidized college education are presented in Figure 8).

In such an educational system, what is the relative importance of individual abilities and financial resources, of parental background and parental income?

According to the solution of the model described above (note, this solution is presented in Figure 3), college admission mainly depends on individual ability level at the moment of college application. The available financial resources do not influence the probability of being admitted unless the direct costs of study \( t \) are relatively high and the financial constraint \( W_i \geq s_i \cdot t \) cuts the left part of the patched region. In this case (with high direct costs of study \( t \)) the available financial resources for education becomes an important factor determining the student’s college admission. As we aim to compare this system with the system, which also provides full-tuition education (in the subsequent sections), we assume here that direct costs of study \( t \) are relatively small (and, thus, the main selection occurs on the college level - admission selection according to the rule \( I_{p1} \)). Parents’ educational background affects abilities and, thus, is positively associated with college attendance. Household income also positively affects abilities, and through this channel also affects college attendance, but does not influence the probability of being admitted by changing available financial resources.

4.3 The case of homogeneous quality of college education with a possible choice between state-subsidized and full-tuition college education.

In this subsection of the model, we assume that there are two types of college education: state-subsidized colleges and colleges on a full-tuition basis. We also assume that the quality of education is the same within all colleges independent of their types.

Studying on a full-tuition basis leads to the additional payment of tuition fees \( T \), which do not vary with individual abilities or initial resources. Individuals have three choices regarding their education: \( s_i = 0 \) corresponds to a secondary education degree, \( s_i = 1 \) corresponds to the choice to attend a state-subsidized college, \( s_i = 2 \) corresponds to the choice to attend a full-tuition college. For simplicity of the model representation we introduce two choice variables: \( s_{1,i} \) is set to 0 for individuals choosing not to attend college on a state-subsidized basis, \( s_{1,i} \) is set to 1 for individuals choosing to attend college on a state-subsidized basis; \( s_{2,i} \) is set to 0 for individuals choosing not to attend college on a full-tuition basis and \( s_{2,i} \) is set to 1 for individuals who are willing and have a capacity to attend college on a full-tuition basis (could attend college on a full-tuition basis if
they are not admitted on a state-subsidized basis). As the only difference between state-
subsidized and full-tuition colleges is the tuition fees payment (recall no differences in the
quality of college education and, thus, in the future returns to a college degree in the labor
market), we can assume the following correspondence between variables $s_{1,i}$ and $s_{2,i}$: for
all $s_{1,i} = 0 \Rightarrow s_{2,i} = 0$ (those who choose not to attend college on a state-subsidized basis
would not attend it on a full-tuition basis), for all $s_{2,i} = 1 \Rightarrow s_{1,i} = 1$ (those who choose to
attend college on a full-tuition basis would also attend it on a state-subsidized basis).

The lifetime value function for non-college attendance of an individual $i$ stays the same
as in the previous subsection (Equation 2). The value function for state-subsidized college
attendance for an individual $i$ can be expressed as the value function for college attendance
in the previous subsection (Equation 3). The value function for college attendance on a
full-tuition basis for an individual $i$ is represented by Equation 9.

$$w_2(\theta_i)$$ represents the wage in the labor market with a college degree obtained in a full-tuition college. In the
current subsection of the model, we assume that $w_2(\theta_i) = w_1(\theta_i)$ for all $\theta_i$.

$$V_{0,i} = U(W_i) + U(w_0(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} \quad (2)$$
$$V_{1,i} = U(W_i - t) + \nu(\theta_i) + [U(w_1(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} - U(w_1(\theta_i))] \quad (3)$$
$$V_{2,i} = U(W_i - t - T) + \nu(\theta_i) + [U(w_2(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} - U(w_2(\theta_i))] \quad (9)$$

Thus, individual $i$ solves the following maximization problem, choosing $s_{1,i} = \{0, 1\}$
and $s_{2,i} = \{0, 1\}$, and, therefore, choosing $s_i = \{0, 1, 2\}$ (where $s_i = s_{1,i} + s_{2,i}$):

$$V_i = \max_{s_i} \{(1 - s_{1,i}) \cdot V_{0,i} ; \quad s_{1,i} \cdot [V_{1,i} \cdot I_p1(\theta_i) + V_{0,i} \cdot (1 - I_p1(\theta_i))] \} ;$$
$$s_{2,i} \cdot [V_{2,i} \cdot I_p2(\theta_i) + V_{0,i} \cdot (1 - I_p2(\theta_i))] \} \}$$

subject to:

$$s_{1,i} \cdot t \leq W_i$$
$$s_{2,i} \cdot (t + T) \leq W_i$$

The level of ability required to receive a positive net lifetime return to college is higher
for full-tuition colleges: $\tilde{\theta}_2 > \tilde{\theta}_1$.

The increase in initial resources (caused by the increase in family income) will lead to a
higher increase in a net lifetime gain from full-tuition college compared to state-subsidized
college (because of the concavity of utility function):
\[ \frac{\partial V(W_i, \theta_i, s_{1,i} = 1)}{\partial W_i} < \frac{\partial V(W_i, \theta_i, s_{2,i} = 1)}{\partial W_i} \]  

(11)

\[ \frac{\partial V(W_i, \theta_i, s_{1,i} = 1)}{\partial W_i} = U'_W(W_i - t) - U'_W(W_i) \]

\[ \frac{\partial V(W_i, \theta_i, s_{2,i} = 1)}{\partial W_i} = U'_W(W_i - t - T) - U'_W(W_i) \]

That is why the difference between \( \tilde{\theta}_2 \) and \( \tilde{\theta}_1 \) will decrease with the increase in the initial resources available for education.

\( I_{p2}(\theta_i) \) represents the admission rule for full-tuition colleges, \( I_{p1}(\theta_i) \) represents the admission rule for state-subsidized colleges. The value of \( I_{p2}(\theta_i) \) is set to 1 if an individual \( i \) who decided to enter a college on a full-tuition basis will be admitted, and \( 0 \) if individual \( i \) will not be admitted. As in the previous subsection, \( \theta_{\text{min,1}} \) and \( \theta_{\text{min,2}} \) are the minimal required levels of ability to be admitted to a state-subsidized and full-tuition college correspondingly. Therefore, for all \( \theta_i \geq \theta_{\text{min,1}} \implies I_{p1}(\theta_i) = 1 \), and for all \( \theta_i < \theta_{\text{min,1}} \implies I_{p1}(\theta_i) = 0 \). The same is true for full-tuition colleges in the tertiary education system: for all \( \theta_i \geq \theta_{\text{min,2}} \implies I_{p2}(\theta_i) = 1 \), and for all \( \theta_i < \theta_{\text{min,2}} \implies I_{p2}(\theta_i) = 0 \). \( \theta_{\text{min,1}} \) and \( \theta_{\text{min,2}} \) are determined on the basis of the number of applications to colleges of each type and the number of available places in those colleges \( M_1 \) and \( M_2 \) correspondingly:

\[ \sum_{i=1}^{N} (I_{p1}(\theta_i)) = M_1 \quad \text{and} \quad \sum_{i=1}^{N} (I_{p2}(\theta_i)) = M_2. \]

Therefore, after receiving students’ applications colleges, both state-subsidized and full-tuition, solve the following maximization problems, determining cut-off ability levels (\( \theta_{\text{min,1}} \) and \( \theta_{\text{min,2}} \) correspondingly) and admission decisions (\( I_{p1}(\theta_i) \) and \( I_{p2}(\theta_i) \)) for each student \( i \). Note that \( I_{p1}(\theta_i) = 0 \) for those who are not applying to state-subsidized colleges, thus, for those with \( s_{1,i} = 0 \); \( I_{p2}(\theta_i) = 0 \) for those who are not applying to full-tuition colleges, thus, for those with \( s_{2,i} = 0 \).

\[ I_1 = \max_{I_{p1}(\theta_i)} \left\{ \sum_{i=1}^{N} (I_{p1}(\theta_i) \cdot \theta_i \cdot s_{1,i}) \right\} \]  

(12)

subject to:

\[ \sum_{i=1}^{N} (I_{p1}(\theta_i)) = M_1 \]
\[ I_2 = \max_{I_{p2}(\theta_i)} \left\{ \sum_{i=1}^{N} (I_{p2}(\theta_i) \cdot \theta_i \cdot \theta_{s_2,i}) \right\} \]  

subject to: 
\[ \sum_{i=1}^{N} (I_{p2}(\theta_i)) = M_2 \]

Figure 4 illustrates the equilibrium in the case of state-subsidized and full-tuition college education assuming that all college education is homogeneous (thus, the solution of maximization problems (10), (12), and (13)).

Figure 4: Homogeneous college education on a state-subsidized or full-tuition basis

As previously, the shaded region (\(0abcde\)) corresponds to the distribution of characteristics \(\theta_i\) and \(W_i\) in the population of secondary school graduates. The area \(gfbedh\) shows individuals with a positive net lifetime gain from state-subsidized college (those who are applying for a state-subsidized college admission). The area \(mlbc\) corresponds to individuals with a positive net lifetime gain from full-tuition college (those who are applying for a full-tuition college admission). Note that those with a positive net lifetime gain from full-tuition college also have a positive and higher lifetime gain from state-subsidized college attendance. The diagonally striped region (\(kbcj\)) illustrates individuals gaining college admission on a state-subsidized basis (\(M_1\) individuals). The region with mesh fill (\(sjpr\)) represents individuals gaining college admission on a full-tuition basis (\(M_2\) individuals).
These two areas are determined according to cut-off rules of college admission - $I_{p1}(\theta_i)$ and $I_{p2}(\theta_i)$ - and, consequently, according to minimal ability level requirements - $\theta_{\text{min},1}$ and $\theta_{\text{min},2}$. Here, the proportion of students with a positive net lifetime gain from college (on a state-subsidized or full-tuition basis) is higher than the number of available places in colleges. This fact also corresponds (as in the previous section) to the empirical evidence for the Russian tertiary education system: differences by federal districts in the proportion of the number of admitted students to the number of students applying to colleges (on a state-subsidized or full-tuition basis) are presented in Table [1].

In such an educational system, what is the relative importance of individual abilities and financial resources, of parental background and parental income?

According to the solution of the model described above (note, this solution is presented in Figure [4]), initial available resources have a higher influence on college admission compared to the case of only state-subsidized colleges. Available financial resources significantly influence the probability of being admitted to full-tuition colleges. As the figure depicts, there are individuals with the same ability level as those admitted to full-tuition colleges (with $\theta_i \geq \theta_{\text{min},2}$, but $\theta_i < \theta_{\text{min},1}$), who do not apply to full-tuition colleges because of the financial constraints. Parents’ educational background affects abilities and, thus, is positively associated with college attendance, as in the previous subsection. Household income, in contrast to the case with only state-subsidized colleges, significantly influences the probability of being admitted to full-tuition colleges not only by affecting students’ abilities, but also by determining available financial resources. Thus, family income plays a significantly greater role in college attendance, compared to the system with only state-subsidized education.

It is worth noting, that if a country passes from the educational system 1 to the educational system 2, and increases college enrollment by adding full-tuition places in colleges (thus, $M_1 = M$ and $M_2 > 0$), students who can afford full-tuition college education and who cannot be admitted to state-subsidized colleges because of low level of ability ($\theta_i < \theta_{\text{min},1} = \theta_{\text{min}}$) would significantly benefit from this change. This change would not affect low-income families who cannot afford full-tuition college education and cannot be admitted to state-subsidized colleges. If a country passes from the educational system 1 to the educational system 2 keeping the number of college students constant ($M_1 + M_2 = M$, and thus, $\theta_{\text{min},2} < \theta_{\text{min}} < \theta_{\text{min},1}$), the places in colleges will be reallocated from high-ability and low-income students ($\theta_{\text{min}} \leq \theta_i < \theta_{\text{min},1}$ and $W_i < t + T$) to low-ability and high-income students ($\theta_{\text{min},2} \leq \theta_i < \theta_{\text{min}}$ and $W_i \geq t + T$).

4.4 The case of heterogeneous quality of college education on a state-subsidized basis.

In this subsection, we relax the assumption of homogeneous quality of college education. We assume that there are two types of state-subsidized college education: high ($H$) and
The value function for state-subsidized high-quality college attendance of an individual is represented by Equation \[ V_{1,H,i} = U(W_i - t) + \nu(\theta_i) + [U(w_{1,H}(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} - U(w_{1,L}(\theta_i))] \] for all \( \theta > \theta_{1,L} \) and \( \theta > \theta_{1,H} \). Abilities affect wages as previously: \( w_s(\theta_1) > w_s(\theta_2) \) for all \( \theta_1 > \theta_2 \).

The lifetime value function for non-college attendance of an individual is represented by Equation \[ V_{0,i} = U(W_i) + U(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} \]

Because of the positive returns to education quality in the labor market, \( V_{1,L,i} < V_{1,H,i} \) for all \( i \). Therefore, the level of ability required to receive a positive net lifetime return to college is higher for low-quality colleges: \( \hat{\theta}_{1,L} > \hat{\theta}_{1,H} \).

Thus, individual \( i \) solves the following maximization problem, choosing \( s_{1L,i} = \{0, 1\} \) and \( s_{1H,i} = \{0, 1\} \). Note that for all individuals \( s_{1H,i} = 0 \Rightarrow s_{1L,i} = 0 \) - those who choose not to apply for high-quality colleges on a state-subsidized basis would not apply for low-quality; for all \( s_{1L,i} = 1 \Rightarrow s_{1H,i} = 1 \) - those who choose to apply for low-quality colleges on a state-subsidized basis would as well apply for high-quality colleges.

\[
V_i = \max_{s_{1L,i},s_{1H,i}} \left\{ (1 - s_{1H,i}) \cdot V_{0,i} ; \ s_{1L,i} \cdot [V_{1,L,i} \cdot I_{p1L}(\theta_i) + V_{0,i} \cdot (1 - I_{p1L}(\theta_i))] ; \ s_{1H,i} \cdot [V_{1,H,i} \cdot I_{p1H}(\theta_i) + V_{0,i} \cdot (1 - I_{p1H}(\theta_i))] \right\} \\
\text{subject to:} \\
s_{1L,i} \cdot t \leq W_i \\
s_{1H,i} \cdot t \leq W_i
\]

The minimal required levels of ability to be admitted to state-subsidized low-quality and high-quality colleges (\( \theta_{\min,1L} \) and \( \theta_{\min,1H} \)) are determined on the basis of the number of applications to colleges of each type and the number of available places in those colleges (\( M_L \) and \( M_H \) correspondingly): \( \sum_{i=1}^{N}(I_{p1L}(\theta_i)) = M_L \) and \( \sum_{i=1}^{N}(I_{p1H}(\theta_i)) = M_H \). As we have seen, a high-quality college is a strictly preferred alternative by all individuals, that is why the high-ability individuals will sort to high-quality colleges, and, therefore, the minimal ability level imposed by low-quality colleges will be lower than those imposed by
high-quality colleges: $\theta_{\min,1L} < \theta_{\min,1H}$.

Thus, colleges of two quality types solve the following maximization problems, determining cut-off ability levels ($\theta_{\min,1L}$ and $\theta_{\min,1H}$) and admission decisions ($I_{p1L}(\theta_i)$ and $I_{p1H}(\theta_i)$) for each student $i$. Note that $I_{p1L}(\theta_i) = 0$ for those who are not applying to state-subsidized low-quality colleges, thus, for those with $s_{1L,i} = 0$; $I_{p1H}(\theta_i) = 0$ for those who are not applying to state-subsidized high-quality colleges, thus, for those with $s_{1H,i} = 0$.

\[ I_{1L} = \max_{I_{p1L}(\theta_i)} \left\{ \sum_{i=1}^{N} (I_{p1L}(\theta_i) \cdot \theta_i \cdot s_{1L,i}) \right\} \quad (17) \]

subject to:
\[ \sum_{i=1}^{N} (I_{p1L}(\theta_i)) = M_L \]

\[ I_{1H} = \max_{I_{p1H}(\theta_i)} \left\{ \sum_{i=1}^{N} (I_{p1H}(\theta_i) \cdot \theta_i \cdot s_{1H,i}) \right\} \quad (18) \]

subject to:
\[ \sum_{i=1}^{N} (I_{p1H}(\theta_i)) = M_H \]

Figure 5 illustrates the equilibrium in the case of state-subsidized college education of two quality levels: high and low (thus, the solution of maximization problems [16], [17] and [18]).

In such an educational system, what is the relative importance of individual abilities and financial resources, of parental background and parental income?

According to the solution of the model described above (Figure 5), college admission mainly depends on individual ability level at the moment of application to colleges. Moreover, the sorting of students between high and low quality colleges is determined only by their abilities. Therefore, parents’ educational background, which affects abilities, is positively associated with both college attendance and quality of college education. Household income also positively affects abilities, and through this channel also affects college attendance and quality of college education, but does not influence the probability of being admitted by changing available financial resources.
4.5 The case of heterogeneous quality of college education on a full-tuition basis.

Before moving to the case with heterogeneous quality of college education and a possible choice between state-subsidized and full-tuition colleges, it is worth looking briefly at what happens if we have only full-tuition colleges with heterogeneous quality of education.

We assume that there are two types of full-tuition college education: high \((H)\) and low \((L)\) quality of education. \(w_{2,H}(\theta_i)\) reflects earnings per period in the labor market of an individual \(i\) with college education of high-quality level \((H)\). \(w_{2,L}(\theta_i)\) reflects earnings per period in the labor market of an individual \(i\) with college education of low-quality level \((L)\). As previously, we assume that the education quality positively influences wages. Thus, \(w_{2,H}(\bar{\theta}) > w_{2,L}(\bar{\theta}) > w_0(\bar{\theta}) > 0\) for all \(\bar{\theta}\). Abilities affect wages as previously: \(w_s(\theta_1) > w_s(\theta_2)\) for all \(\theta_1 > \theta_2\). We also assume that high-quality education is more costly, than the low-quality education: \(T_H > T_L > 0\).

The lifetime value function for non-college attendance of an individual \(i\) stays the same as in the previous subsections (Equation 2). The value function for full-tuition low-quality college attendance of an individual \(i\) can be expressed according to Equation 19. The value function for full-tuition high-quality college attendance of an individual \(i\) is represented by Equation 20.
\[ V_{0,i} = U(W_i) + U(w_0(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} \]  

\[ V_{2L,i} = U(W_i - t - T_L) + \nu(\theta_i) + [U(w_{2L}(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} - U(w_{2L}(\theta_i))] \]  

\[ V_{2H,i} = U(W_i - t - T_H) + \nu(\theta_i) + [U(w_{2H}(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} - U(w_{2H}(\theta_i))] \]  

\[ \tilde{\theta}_{2L} \] is the minimal level of ability required to receive a positive net lifetime return to a low-quality college and \( \tilde{\theta}_{2H} \) is the minimal level of ability required to receive a positive net lifetime return to a high-quality college. Without additional assumption about the form of wages and tuition fees we cannot define a-priori whether \( \tilde{\theta}_{2L} \leq \tilde{\theta}_{2H} \). But, as in the previous sections \( \tilde{\theta}_{2L} \) and \( \tilde{\theta}_{2H} \) are decreasing with greater initial financial resources (because of the concavity of utility function). Though there is a positive return to education quality in the labor market, but as high-quality education is more costly, there is no strict preferences of the students towards the high-quality or lower-quality colleges (we cannot define whether \( V_{2H,i} \leq V_{2L,i} \)). Individual \( i \) prefers high-quality education to low-quality education if:

\[ V_{2H,i} - V_{2L,i} = \{ U(W_i - t - T_H) - U(W_i - t - T_L) \} + \]

\[ + [U(w_{2H}(\theta_i)) - U(w_{2L}(\theta_i))] \cdot \frac{\beta - \beta^T}{1 - \beta} > 0 \]  

The first part of the equation \( (U(W_i - t - T_H) - U(W_i - t - T_L)) \) is negative and decreasing in absolute value with increasing initial financial resources. The second part of the equation \( (U(w_{2H}(\theta_i)) - U(w_{2L}(\theta_i))) \) is positive and increasing with the rise in ability level (we assume that \( \{ U'_{w_{2H}} \cdot w'_{2H}(\theta_i) - U'_{w_{2L}} \cdot w'_{2L}(\theta_i) \} > 0 \) ). Thus, there exists a level of ability \( \tilde{\theta}_{2H vs L} \) so that for all \( i \) with \( \theta_i \geq \tilde{\theta}_{2H vs L} \) a net lifetime gain from high-quality college is higher than a net lifetime gain from low-quality college. \( U(W_i - t - T_H) - U(W_i - t - T_L) \) is negative and decreasing in absolute value with increasing initial financial resources, thus, \( \tilde{\theta}_{2H vs L} \) is decreasing with a rise in initial financial resources. Formal proof of this fact is presented below.
\begin{equation}
V_{2H,i}(\tilde{\theta}_{2,HvsL}) - V_{2L,i}(\tilde{\theta}_{2,HvsL}) = 0
\end{equation}

\begin{equation}
U(W_i - t - T_H) - U(W_i - t - T_L) +
\{U(w_{2,H}(\tilde{\theta}_{2,HvsL})) - U(w_{2,L}(\tilde{\theta}_{2,HvsL}))\} \cdot \frac{\beta - \beta^T}{1 - \beta} = 0
\end{equation}

thus:
\begin{equation}
\frac{\partial(\tilde{\theta}_{2,HvsL})}{\partial(W_i)} = - \frac{U'(W_i - t - T_H) - U'(W_i - t - T_L)}{(U'_{w_{2,H}}(\tilde{\theta}_{2,HvsL}) - U'_{w_{2,L}}(\tilde{\theta}_{2,HvsL})) \cdot \frac{\beta - \beta^T}{1 - \beta}} < 0
\end{equation}

as:
\begin{align*}
U'(W_i - t - T_H) - U'(W_i - t - T_L) &> 0 \\
(U'_{w_{2,H}}(\tilde{\theta}_{2,HvsL}) - U'_{w_{2,L}}(\tilde{\theta}_{2,HvsL})) \cdot \frac{\beta - \beta^T}{1 - \beta} &> 0
\end{align*}

Individual $i$ solves the following maximization problem, choosing $s_{2L,i} = \{0, 1\}$ and $s_{2H,i} = \{0, 1\}$:
\begin{align*}
V_i = \max_{s_{2L,i}, s_{2H,i}} & \{(1 - \max\{s_{2H,i}; s_{2L,i}\}) \cdot V_{0,i} + s_{2L,i} \cdot [V_{2L,i} \cdot I_{p2L}(\theta_i) + V_{0,i} \cdot (1 - I_{p2L}(\theta_i))] + s_{2H,i} \cdot [V_{2H,i} \cdot I_{p2H}(\theta_i) + V_{0,i} \cdot (1 - I_{p2H}(\theta_i))] \} \\
\text{subject to:} & \quad s_{2L,i} \cdot (t + T_L) \leq W_i \\
& \quad s_{2H,i} \cdot (t + T_H) \leq W_i
\end{align*}

Thus, colleges solve the following maximization problems, determining cut-off ability levels ($\theta_{min,2L}$, $\theta_{min,2H}$) and admission decisions ($I_{p2L}(\theta_i)$, $I_{p2H}(\theta_i)$) for each student $i$.
\begin{align*}
I_{2L} = \max_{I_{p2L}(\theta_i)} & \left\{ \sum_{i=1}^{N} (I_{p2L}(\theta_i) \cdot \theta_i \cdot s_{2L,i}) \right\} \\
\text{subject to:} & \quad \sum_{i=1}^{N} (I_{p2L}(\theta_i)) = M_L
\end{align*}

\begin{align*}
I_{2H} = \max_{I_{p2H}(\theta_i)} & \left\{ \sum_{i=1}^{N} (I_{p2H}(\theta_i) \cdot \theta_i \cdot s_{2H,i}) \right\} \\
\text{subject to:} & \quad \sum_{i=1}^{N} (I_{p2H}(\theta_i)) = M_H
\end{align*}

Note that $I_{p2L}(\theta_i) = 0$ for those who are not applying to full-tuition low-quality colleges.
thus, for those with \( s_{2L,i} = 0 \); \( I_{p2H}(\theta_i) = 0 \) for those who are not applying to full-tuition high-quality colleges, thus, for those with \( s_{2H,i} = 0 \).

*Figure 6* illustrates the equilibrium in the case of full-tuition college education with two quality levels (high and low) and correspondingly with two levels of tuition fees (high and low); thus, the solution of maximization problems 23, 24 and 25.

The minimal required levels of ability to be admitted to full-tuition low-quality and high-quality colleges (\( \theta_{\text{min},2L} \) and \( \theta_{\text{min},2H} \)) are determined on the basis of the number of applications to colleges of each type and the number of available places in those colleges \((M_L \text{ and } M_H \text{ correspondingly})\). As can be seen in Figure 6, \( \theta_{\text{min},2L} \) may be higher or lower than \( \theta_{\text{min},2L} \) - their relative values depend on the proportion of the population with desire and capacity to pay full-tuition fees for high-quality colleges and on the number of available places in colleges (Figure 6 1st and 2nd graphs). Moreover, sorting of students among high-quality and low-quality colleges is determined by the ability level \( \bar{\theta}_{2,HvsL} \), which depends on the differences in tuition costs and returns to college education between high-quality and low-quality colleges. Thus, the 3rd graph of the Figure 6 illustrates the case of the high level of \( \bar{\theta}_{2,HvsL} \), which leads to that some students even with a high level of financial resources (enough to pay for a high-quality college) choose low-quality colleges (area xyz).

In such an educational system, what is the relative importance of individual abilities and financial resources, of parental background and parental income?

According to the solution of the model described above (Figure 6), college admission mainly depends on individual ability level at the moment of college application. However, the sorting of students between high and low quality colleges is determined mainly by financial resources of their parents. Therefore, parents’ educational background, which affects abilities, is positively associated with college attendance and with quality of college education. Moreover, household income influences college admission and college quality by determining available financial resources and by influencing individual lifetime value functions of high-quality and low-quality colleges (thus, affecting the level of ability determining preferences between high and low education quality - \( \bar{\theta}_{2,HvsL} \)).

A comparison between the educational system 4 and the educational system 3 (full-tuition colleges and state-subsidized colleges with heterogenous education) suggests that in the case of full-tuition college education the sorting among high-quality and low-quality colleges is more determined by financial resources than it is in the case of state-subsidized education. Thus, within a full-tuition educational system, family income, compared to parents’ education, might have a stronger effect on the sorting of students among colleges with different quality of education. In the first case (with state-subsidized colleges) parents’ educational background has significantly higher influence on the sorting of students among colleges with different educational qualities, than household financial resources.
Figure 6: Heterogeneous college education on a full-tuition basis
4.6 The case of heterogeneous quality of college education with a possible choice between state-subsidized and full-tuition college education.

In this final subsection of the model, we assume that there are two types of state-subsidized college education and two types of full-tuition college education: colleges with high (\(H\)) and low (\(L\)) quality of education.

The lifetime value functions for the five possible educational choices can be expressed by equations 2, 14, 15, 19, 20. \(V_{0,i}\) represents the lifetime value function for non-attendance of any college. \(V_{1L,i}\) represents the lifetime value function for low-quality college attendance on a state-subsidized basis. \(V_{1H,i}\) represents the lifetime value function for high-quality college attendance on a state-subsidized basis. \(V_{2L,i}\) represents the lifetime value function for low-quality college attendance on a full-tuition basis. \(V_{2H,i}\) represents the lifetime value function for high-quality college attendance on a full-tuition basis.

\[
V_{0,i} = U(W_i) + U(w_0(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} \quad \text{(2)}
\]

\[
V_{1L,i} = U(W_i - t) + \nu(\theta_i) + [U(w_{1L}(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} - U(w_{1L}(\theta_i))]
\]

\[
V_{1H,i} = U(W_i - t) + \nu(\theta_i) + [U(w_{1H}(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} - U(w_{1H}(\theta_i))]
\]

\[
V_{2L,i} = U(W_i - t - T_L) + \nu(\theta_i) + [U(w_{2L}(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} - U(w_{2L}(\theta_i))]
\]

\[
V_{2H,i} = U(W_i - t - T_H) + \nu(\theta_i) + [U(w_{2H}(\theta_i)) \cdot \frac{1 - \beta^T}{1 - \beta} - U(w_{2H}(\theta_i))]
\]

We assume that in the labor market: \(w_{1L}(\theta_i) = w_{2L}(\theta_i) < w_{1H}(\theta_i) = w_{2H}(\theta_i)\).

Thus, individual \(i\) solves the following maximization problem, choosing \(s_{1L,i} = \{0, 1\}\), \(s_{1H,i} = \{0, 1\}\), \(s_{2L,i} = \{0, 1\}\), and \(s_{2H,i} = \{0, 1\}\). Note that for all individuals \(s_{1H,i} = 0 \Rightarrow s_{1L,i} = 0\) - those who choose not to apply for high-quality state-subsidized college would not apply for low-quality; for all \(s_{1L,i} = 1 \Rightarrow s_{1H,i} = 1\) - those who choose to apply for low-quality colleges on a state-subsidized basis would as well apply for high-quality colleges on a state-subsidized basis; for all \(s_{1H,i} = 0 \Rightarrow s_{2H,i} = 0\) and \(s_{1L,i} = 0 \Rightarrow s_{2L,i} = 0\) (those who choose not to attend college on state-subsidized basis would not attend it on a full-tuition basis), for all \(s_{2H,i} = 1 \Rightarrow s_{1H,i} = 1\) and \(s_{2L,i} = 1 \Rightarrow s_{1L,i} = 1\) (those who choose to attend college on a full-tuition basis would also attend it on a state-subsidized basis).
\[ V_i = \max_{s_{1L,i}, s_{1H,i}, s_{2L,i}, s_{2H,i}} \left\{ (1 - s_{1H,i}) \cdot V_{0,i} \right\} \]

subject to:

\[ s_{1L,i} \cdot t \leq W_i; \quad s_{1H,i} \cdot t \leq W_i; \quad s_{2L,i} \cdot (t + T_L) \leq W_i; \quad s_{2H,i} \cdot (t + T_H) \leq W_i. \]  

Thus, colleges solve the following maximization problems, determining cut-off ability levels \((\theta_{\text{min,}1L}, \theta_{\text{min,}1H}, \theta_{\text{min,}2L}, \theta_{\text{min,}2H})\) and admission decisions \((I_{p1L}(\theta_i), I_{p1H}(\theta_i), I_{p2L}(\theta_i), I_{p2H}(\theta_i))\) for each student \(i\).

\[ I_{1L} = \max_{I_{p1L}(\theta_i)} \left\{ \sum_{i=1}^{N} (I_{p1L}(\theta_i) \cdot \theta_i \cdot s_{1L,i}) \right\} \]

subject to:

\[ \sum_{i=1}^{N} (I_{p1L}(\theta_i)) = M_{L1} \]  

\[ I_{1H} = \max_{I_{p1H}(\theta_i)} \left\{ \sum_{i=1}^{N} (I_{p1H}(\theta_i) \cdot \theta_i \cdot s_{1H,i}) \right\} \]

subject to:

\[ \sum_{i=1}^{N} (I_{p1H}(\theta_i)) = M_{H1} \]  

\[ I_{2L} = \max_{I_{p2L}(\theta_i)} \left\{ \sum_{i=1}^{N} (I_{p2L}(\theta_i) \cdot \theta_i \cdot s_{2L,i}) \right\} \]

subject to:

\[ \sum_{i=1}^{N} (I_{p2L}(\theta_i)) = M_{L2} \]
\[ I_{2H} = \max_{I_{p2H}(\theta_i)} \left\{ \sum_{i=1}^{N} (I_{p2H}(\theta_i) \cdot \theta_i \cdot s_{2H,i}) \right\} \tag{30} \]

subject to:
\[ \sum_{i=1}^{N} (I_{p2H}(\theta_i)) = M_{H,2} \]

Note that \( I_{p2L}(\theta_i) = 0 \) for those who are not applying to full-tuition low-quality colleges, thus, for those with \( s_{2L,i} = 0 \); \( I_{p2H}(\theta_i) = 0 \) for those who are not applying to full-tuition high-quality colleges, thus, for those with \( s_{2H,i} = 0 \). \( I_{p1L}(\theta_i) = 0 \) for those who are not applying to state-subsidized low-quality colleges, thus, for those with \( s_{1L,i} = 0 \); \( I_{p1H}(\theta_i) = 0 \) for those who are not applying to state-subsidized high-quality colleges, thus, for those with \( s_{1H,i} = 0 \).

Figure 7 illustrates the solution of the maximization problems 26, 27, 28, 29, and 30 — the equilibrium in the case of state-subsidized and full-tuition colleges of two quality levels (high and low) and correspondingly of two levels of tuition fees (high and low).

The minimal required levels of ability to be admitted to full-tuition low-quality and high-quality colleges (\( \theta_{\min,2L} \) and \( \theta_{\min,2H} \)), as well as the minimal required levels of ability to be admitted to state-subsidized low-quality and high-quality colleges (\( \theta_{\min,1L} \) and \( \theta_{\min,1H} \)) are determined as the solutions of the maximization problems 27, 28, 29, and 30 on the basis of the number of applications to colleges of each type and the number of available places in those colleges (\( M_{L,2} \), \( M_{H,2} \), \( M_{L,1} \), and \( M_{H,1} \) correspondingly). The cut-off rules applied by colleges are the following: \( I_{p2L}(\theta_i) \), \( I_{p2H}(\theta_i) \), \( I_{p1L}(\theta_i) \), and \( I_{p1H}(\theta_i) \).

The alternative of attending a state-subsidized high-quality college is better than all other college possibilities, thus, \( \theta_{\min,1H} > \theta_{\min,1L} \), \( \theta_{\min,1H} > \theta_{\min,2H} \), and \( \theta_{\min,1H} > \theta_{\min,2L} \). The alternative of attending a low-quality state-subsidized college is strongly preferred to the alternative to attend the low-quality full-tuition college by all individuals, thus \( \theta_{\min,1L} > \theta_{\min,2L} \). We cannot say anything about the relative values of other cut-off rules: \( \theta_{\min,1L} \leq \theta_{\min,2H} \geq \theta_{\min,2L} \). Their relative values depend on the proportion of the population with desire and capacity to pay full-tuition fees for high/low-quality colleges and on the number of available places in colleges. Moreover, as in the previous subsection, sorting of students among high-quality and low-quality full-tuition colleges is determined by the ability level \( \bar{\theta}_{2Hvs2L} \), which depends on the differences in tuition costs and returns to college education between high-quality and low-quality colleges. Sorting of students (who have the capacity to pay for high-quality full-tuition college) among high-quality full-tuition and low-quality state-subsidized colleges is determined by the ability level \( \bar{\theta}_{2Hvs1L} \), which also depends on the tuition costs and on the differences in returns to college education between high-quality and low-quality colleges. Note that \( \bar{\theta}_{2Hvs1L} \geq \bar{\theta}_{2Hvs2L} \geq \bar{\theta}_{2H} > \bar{\theta}_{1H} \), \( \bar{\theta}_{2L} \geq \bar{\theta}_{1L} \), and these levels are decreasing with a rise in initial financial resources. For
Figure 7: Heterogeneous college education on a state-subsidized and full-tuition basis
a simplicity of the graphic representation we do not illustrate cases where $\tilde{\theta}_{2Hvs2L}$ and $\tilde{\theta}_{2Hvs1L}$ cut the patched regions, but one can easily imagine the consequent distribution of students among analyzed college types, taking as an example the 3rd graph of Figure 6 in the previous subsection. However, Figure 7 represents two possible cases: when in the equilibrium $\theta_{\text{min},1L} < \theta_{\text{min},2H}$ and when in the equilibrium $\theta_{\text{min},1L} > \theta_{\text{min},2H}$.

In such an educational system, what is the relative importance of individual abilities and financial resources, of parental background and parental income?

According to the solution of the model described above (Figure 7), college admission in general mainly depends on individual ability level. Nevertheless, the sorting of students between high and low quality colleges is determined to a large extent by financial resources of their parents. Thus, because of the higher or lower family income, some students with the given ability level go to the low-quality state-subsidized colleges and others, with the same ability level, obtain their education in the high-quality full-tuition colleges (concerned students are those with the ability level $\theta_i$ such as $\text{max}\{\theta_{\text{min},2H};\theta_{\text{min},1L}\} \leq \theta_i < \theta_{\text{min},1H}$). Therefore, parents’ educational background, which affects abilities, is positively associated with college attendance and with quality of college education. Moreover, household income significantly influences college quality by determining available financial resources and by influencing individual lifetime value functions of high-quality and low-quality colleges (thus, affecting the level of ability determining preferences between high and low education quality - $\tilde{\theta}_{2Hvs1L}$ and $\tilde{\theta}_{2Hvs2L}$).

It is worth comparing the transition from the educational system 3 (only state-subsidized colleges with two quality levels of education) to the educational system 5 described in this subsection. Let us look at the case if a country passes from the educational system 3 to the educational system 5 and does not increase total college enrollment (thus, $M_{H,1} + M_{H,2} = M_H$ and $M_{L,1} + M_{L,2} = M_L$). In this case, high-income students would be reallocated from low-quality state-subsidized colleges to high-quality full-tuition colleges. Low-income students would have to move from high-quality colleges to low-quality state-subsidized colleges (besides those who have the highest ability levels in the population) and some of them would not go to colleges at all (students with lowest abilities and income among those admitted within the educational system 3). Therefore, high-income students would gain from this transition in both college enrollment and college quality, but at the expense of low-income students. In this case, household income would determine to a larger extent the quality of college education than parents’ educational background.

Let assume that a country passes from the educational system 3 to the educational system 5 and increases the number of college students. If we add only low-quality full-tuition colleges ($M_{L,1} = M_L$ and $M_{L,2} > 0$), a larger number of high-income students would be able to enroll in colleges (those who cannot be admitted to state-subsidized colleges and who can afford full-tuition college education). This change would not influence low-income students who cannot be admitted to state-subsidized colleges. In this case household
income, as well as parents’ educational background, would influence the college enrollment probabilities.

If we add high-quality full-tuition colleges to the educational system \( 3 \) \( (M_{H,1} = M_H \) and \( M_{H,2} > 0) \), a larger number of high-income students would be enrolled in colleges. Moreover, high-income students would move from low-quality state-subsidized colleges to high-quality full-tuition colleges. The influence of household income on college quality would increase considerably. At the same time, and it is worth underlying this fact, a larger number of low-income students would be able to obtain college education (thought it would be low-quality college education). Low-income students with lower levels of ability (lower ability levels compared to those that would be admitted to colleges with the educational system \( 3 \) ) would be able to enroll in low-quality colleges on the places of high-income students who moved to high-quality full-tuition colleges.

Therefore, the transition from the educational system \( 3 \) (only state-subsidized college education) to the educational system \( 5 \) (with college education on a state-subsidized and on a full-tuition basis) keeping college enrollment constant would provide an advantage to high-income students (in terms of college enrollment and college quality) at the expense of low-income students, and, thus, would increase the role of household income in college admission and college quality choice. On the other hand, the transition from the educational system \( 3 \) to the educational system \( 5 \), which increases the number of college students, would benefit both high-income and low-income students in terms of college enrollment. Nevertheless, household income would have a stronger effect on the quality of college education compared to parents’ educational background.
5 Empirical Model

this section is under development

6 Results

this section is under development

7 Conclusion

this section is under development
References


Table 1: The proportion of the number of admitted students to the number of students applying to colleges on a state-subsidized or full-tuition basis.

<table>
<thead>
<tr>
<th>Region</th>
<th>Only on a state-subsidized basis</th>
<th>Including full-tuition and state-subsidized places</th>
</tr>
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<tbody>
<tr>
<td>Russian Federation</td>
<td>2.9</td>
<td>1.9</td>
</tr>
<tr>
<td>Central Region</td>
<td>2.7</td>
<td>1.8</td>
</tr>
<tr>
<td>North-West Region</td>
<td>3.0</td>
<td>2.0</td>
</tr>
<tr>
<td>South Region</td>
<td>2.4</td>
<td>1.8</td>
</tr>
<tr>
<td>Volga Region</td>
<td>3.3</td>
<td>2.0</td>
</tr>
<tr>
<td>Ural Region</td>
<td>3.0</td>
<td>1.8</td>
</tr>
<tr>
<td>Siberia Region</td>
<td>3.1</td>
<td>1.9</td>
</tr>
<tr>
<td>Far-East Region</td>
<td>3.0</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Note: The number of students applying to colleges, which is reported in this table, overstates the true value of the number of applying students, as it includes students that are applying to more than one college. That is why the calculated admission rates (proportion of the number of admitted students to the number of students applying to colleges) might overstate the true rates.

Figure 8: The ratio of the number of admitted students to the number of applying students: state-subsidized colleges in Russia, 2005