Abstract
In this ongoing project, we attempt to integrate the recent developments in the analysis of household labor supply into a search model of marriage. Following Shimer and Smith (2000), matching on the marriage market is assumed time-intensive and, as in the original model of Becker (1973), is driven by output from the match. We explicitly introduce labor supply in the model by (i) linking the output from the match to the productivities (wage) of the partners and (ii) assuming collective labor supply inside the resulting household. Based on numerical simulations, the theoretical analysis assesses whether the standard result of positive assortative matching still holds when the output from the match is both endogenous and non-transferable. We plan to specify and estimate an econometric model of search to test the predictions. The data to be used come from the 1996 wave of the SIPP. By reconciling those two strands of literature, our model should account for some well-documented stylized facts on both marriage patterns – such as the declining age gap between spouses, the rise in wage inequalities inside households or changes in home production technologies – and recent trends in the labor market – e.g. changes in female labour supply and gender wage gap. Policy implications range from the economic consequences of, among others, active policies aimed at sharing care responsibilities for children, targeting social benefits on one spouse rather than the household as a whole or promoting female labour supply.

Keywords: Marriage search model, Collective labor supply, Structural estimation.

JEL classification: C78, D83, J12, J22.

1 Introduction
Since Pencavel (1998)’s claim that “there is an extensive research literature on the market work decisions of husbands and wives, and there is a smaller literature on the marital choices that match...
husbands and wives [but] rarely have these two classes of research been joined”, the respective trends of those two strands of research have bended to more convergence, though without crossing yet.

The so-called collective model of intra-household’s labour market behavior (Chiappori, 1988, 1992) has deeply updated the economic analysis of inside households’ labour supply decisions (see Clark, Couprie, and Sofer, 2004 for a survey in French and Donni, 2008 for a short survey in english). The main contribution of the model is that to show that intra-household substitutions and income sharing crucially affects labour supply decisions. This imped looking at the household as a single decision-unit, as implicitly assumed in the “unitary model”. Indeed, the collective model empirically outperforms the unitary vision of household labour supply (Fortin and Lacroix, 1997). Chiappori, Fortin, and Lacroix (2002) moreover show that the model imposes restrictions that allow to estimate labour supply functions and to identify the parameters governing intra-households income sharing decisions.

The core collective model has recently been extended to include non-convex budget sets (in the form of non-participation and income taxation, Donni, 2003), public consumption induced by the presence of children (Blundell, Chiappori, and Meghir, 2005), as well as heterogeneity and non-participation to the labor market (Blundell, Chiappori, Magnac, and Meghir, 2007). Although those contributions make important steps in widening our understanding of the intra-household decision process, they all still rely on the implicit assumption of exogenously formed households.

Since Becker (1973)’s seminal work on marriage market, the idea that household formation is endogenous to individual traits, giving rise to a correlation between spouses traits – often called assortative matching – has however become commonly admitted. As a result, it nowadays seems intuitive that observed trends in the marriage market and labor market behavior are closely correlated. To give an example, Stevenson and Wolfers (2007) relate the “huge rise in divorce rates that slows down in the last quarter century, rising age of first marriage associated with increasing remarriage rates and declining age gap between spouses” in the last 150 years to the “sharp changes in wage structure, including a rise in inequality and partial closing of the gender wage gap; dramatic changes in home production technologies; and the emergence of the internet as a new matching technology”. Although this lines of reasoning make sense to a labor economist, we’re aware of no formal analysis supporting such claims.

On the labor economics’s side, relating labor supply to the matching decisions on the marriage market could help explain some still controversial stylized facts. One of them is the wage structure and labor market decisions of dual-earners couples. While Stroh and Brett (1996) highlight a dual-earner dad penalty in wages, Jacobsen and Rayack (1996) document a working spouse wage premium. Based on a reduced form model of positive assortative matching on spouses’ earnings, Song (2007) suggests that such differences between working spouse and singles could unduly be attributed to intra-household substitutions when the matching process is ignored. The premium/penalty is rather explained by the correlation in earnings arising from mating decisions. Similar biased conclusion could be drawn by focusing only on intra-household bargaining while evaluating, e.g., the public policies of most OECD
countries towards promoting female labor supply (see OECD 2007 for a recent survey). Accurate evaluations of the impact of such policies on both males’ and females’ labor supply require a structural formalization of the full decision process, including intra-household bargaining conditional on its composition and the endogenous formation of households.

In this paper, we present the current state of an ongoing project aimed at integrating endogenous labor supply decisions into a search model of marriage. The existing theoretical and empirical literature devoted to the marriage market is presented in the next two subsections. In Section 2 we present our model and results from simulation. The empirical analysis will be performed in the next few months, using data we describe in Section 3.1.

1.1 Background literature – Endogenous household formation: Theory

The seminal papers of Becker (1973, 1974) study the matching between heterogeneous spouses on a frictionless marriage market. Individuals are characterized by an exogenous productivity type drawn from a continuous set \( X \). The decision of whom to be married with is driven by the output for spouses of type \( x \) and \( y \) from being married together, \( f(x, y) \). Marriage is assumed to dominate single situations \( (f(x, y) > f(x, 0) + f(0, y)) \), so that marriage is always preferable; and the production function is symmetric.

Within this world of perfect information, Assortative Matching (i.e. marriage between homogeneous spouses, AM) arises if the production function is supermodular, i.e.: \( f(x', y) + f(x, y') > f(x', y') + f(x, y) \) \( \forall x' > x, y > y' \) where \( x, x', y, y' \in X \) – meaning that productivities are complement in terms of marriage output at increasing rate. The results stems from the fact that every heterogeneous matching admits Pareto-improving reshuffling between spouses. The only allocation exhibiting no incentive to individually deviate is the one that maximizes the sum of joint outputs from all marriages, rather than the output for a single couple. Equilibrium matching hence optimally allocates the scarce resource – desirable traits. Empirical predictions based on the model vary with how the characteristic affects the household output. Becker argues that the division of labor in household production makes wage rates substitutes: the lower one spouse’s wage as regards to the other, the less costly it is he/she spends time at home rather than at work (Becker 1973 p.328).

Shimer and Smith (2000) extend Becker’s model to account for imperfect information. Search for a mate is costly due to imperfect knowledge about the unmatched population – to which potential mates belong by assumption. The authors prove the existence of an equilibrium steady state on such a marriage market. An interesting feature of this kind of model is that individual’s strategies are based on matching sets rather than singletons. This results in a feature well-suited to empirical applications, namely that “mismatch is the rule”: whatever the sorting rule that prevails for a given characteristic (assortative or not, positive or negative, . . . ) observed matching between traits is noisy.
A few theoretical papers further build on this seminal analysis. Chen (2005) provides some alternative proofs to Shimer and Smith (2000)’s main results. In the very same framework, Atakan (2006) explicitly models search costs – rather than modeling search frictions as time costs that result from discounting. While Shimer and Smith show that the equilibrium does not necessarily involve assortative matching – even when the production function is supermodular – Atakan’s model provides sufficient conditions that restore the classical result on AM. Cornelius (2003) further widens the empirical relevance of the model by incorporating “on-the marriage search”, hence making separation rate endogenous. Extensions to more specific issues include discussions on either the age of marriage or marriage market institutions. Giolito (2005) focuses on differences in fecundity as a determinant of the age of marriage, while Coles and Francesconi (2007) relies on random success on the labor market to explain age differences between spouses (“toyboy marriages”). Regarding institutions, Mumcu and Saglam (2006) present simulation of a two-period model of matching aimed at forecasting the effects of alimony rate, legal cost of divorce, initial endowments, couple and single productivity parameters on the payoffs and marital status in the society. Fernandez, Guner, and Knowles (2005) makes the most important step towards closing the gap between household formation and spouses’ labor market behavior. The paper examines the interactions between household formation, inequality, and per capita income. Agents decide to become skilled or unskilled and form households. In equilibrium, the degree of marital sorting by skill type and wage inequality is positively correlated across steady states and negatively correlated with per capita income.

This whole strand of literature rely on transferable utility, in the sense that no constraint is imposed on the way the output from the match is shared between spouses. In the context of household consumption and collective labor supply, utility however do has to be non-transferable between spouses. Two reasons for that are the consumption of some common public goods and the individual-specific productivities underlying labor supply choices. While Burdett and Coles (1997) and Adachi (2003) provide a general discussion on existence issues, Legros and Newman (2007) are to our knowledge the first to characterize the equilibrium matching pattern in such a context. They show that matching correlations in traits results not only from complementarity in types but also from complementarity between an agent’s type and his partner’s payoff. We further generalize the approach by highlighting the endogenous decision process that results in such non-transferabilities.

1.2 Background literature – Endogenous household formation: Empirical analysis

The empirical literature dealing with household formation is rather scarce. As far as we know, three papers offer a direct empirical test of Becker’s results – although numerous papers focus on its empirical implications, see Choo and Siow (2006) p.175. Bergstrom and Lam (1994) develop a model of equi-
librium marriage assignment driven by a gender– and age–specific ideal age of marriage. Individuals only care about when (not who) they marry. The model is estimated by maximizing the total output of marriages. Identification of the model relies on variations in fertility during the century.

Suen and Lui (1999) offer a structural estimation of Becker’s model. The production function is assumed linear in individual characteristics – including interaction terms. The authors show that optimal assignment between spouses leads to a (Type I) Tobit on the value of the match between spouses i and j – the value is left-censored when individual perspectives are higher than the output from matching. The empirical results confirm positive assortative matching on wages. The structural model used by Choo and Siow (2006) has the advantage of relying on a non-parametric matching function. The resulting CMNL allows them to recover the value of observed matches as well as the net value of individual gains (i.e. surplus from marriage minus production from being single). The main focus is on how the gain from marriage has changed during the period under study (1971–1981). In particular, legal abortion is shown (from DD estimations) to decrease the gain from marriage.

Empirical search models are now a common feature of labour economics (see, e.g. Postel-Vinay and Robin, 2002a,b and Eckstein and van den Berg, 2007 for a recent survey). Very few works offer an application to a mate search. Earlier works include Rosenzweig and Stark (1989), who study matching – through migrations – in developing countries, where marriage is driven by the expected profits stemming from joint-production in farms. More recently, Bisin, Topa, and Verdier (2004) focus on the religious intermarriage rates implied by a dynamic search model of marriage with biased preferences in favor of own religious beliefs. The search process is shown to distort the socialization rate as compared to what one would expect from exogenous matching. Wong (2003) is the first to perform a structural estimation of the basic marriage model with search based on individual traits related to the labor market. Wage appears to be more desirable than education in predicting marriage opportunities of men. Similar asymmetries according to gender are obtained by Fisman, Iyengar, Kamenica, and Simonson (2006), based on meeting data – rather than matching only. The authors use a natural experiment to estimate the sensibility of the matching process to individual covariates. Women seem to put greater weight on intelligence and race than men do; while men place more value on physical appearance.

Two important limitations of this existing empirical literature are that utility is assumed transferable, hence ruling out any kind of substitutions inside the household, and output from the matched is arbitrarily related to individual traits. We rather consider a collective model of household decision involving consumption of a public good, hence leading to an endogenous, non-transferable output from the match.

(Also see Brien, Lillard, and Stern (2006); Gemici and Laufer (2009).)
2 Theoretical model

The original accounting for heterogeneity between partners in a structural search model is due to Lu and McAfee (1996). Shimer and Smith (2000); Smith (2006) map this framework with the particular case of matching on the marriage market, hence extending the standard model of marriage (Becker, 1973, 1974) to an imperfect information context. We further generalize the approach by accounting for the leisure/consumption trade-off inside potential households, formalized as a collective model of labour supply (Chiappori, 1992).

2.1 Basic set up

We consider an economy made up of $L_m$ males and $L_f$ females, looking for a partner on the marriage market. The number of married couples (i.e. the number of matched people) in the population is denoted $N$. We impose the constraint that a couple necessarily involves one male and one female.

Denoting $U_m$ the number of single males and $U_f$ the number of single females, the marriage market supply functions result from simple demographic accounting equations:

$$U_m = L_m - N \quad (1)$$

$$U_f = L_f - N \quad (2)$$

Individuals are characterized by their labor productivity, denoted $x$ for males, $y$ for females. Assuming a competitive labor market, productivities stand as well for hourly wages. Marriage market is imperfect, however, and only the distribution of wages in the population of males and females are common knowledge. We denote $\ell_m(x)$ and $\ell_f(y)$ the respective measures of x-type males and y-type females in the population, so that $\ell_m(x)/L_m$ and $\ell_f(y)/L_f$ are the corresponding densities of wage in the population. We consider an endogenous matching on those traits, driven by the expected payoff stemming from the selected partner. Matching hence induces a distortion in the distribution of wages according to marital status, and the measures of wages in the population of singles are denoted $u_m$ and $u_f$. Couple formation is based on partners’ types, $(x, y)$, resulting in a measure of matches of type $(x, y)$ denoted $h(x, y)$.

2.2 Matching process

Due to the assumption of heterosexual matches, the tension on the marriage market is $\theta = \frac{U_f}{U_m}$. We moreover rule out “on the marriage-search”, assuming that only singles look for a partner. The number of meetings per period is hence measured by the matching function $M(U_m, U_f)$. $\lambda_i = \frac{M(U_m, U_f)}{U_i}$ is hence the instantaneous probability that a searching individual of gender $i$ meets a possible partner.
A pure strategy for an individual of type $x$ is a set of agents $A_m(y)$ with whom $x$ is willing to match. Similarly, $y$ agents choose $A_f(x)$. The set of admissible matches, $\mathcal{M}$ is the intersection of agents’ strategies, gathering the $(x, y)$ couples formed would an $x$-male and a $y$-female meet each other. Last, matches are exogenously dissolved with instantaneous probability $\delta$. Would a match be destroyed, both partners become singles, hence entering the pool of searching individuals.

**Flow equations.** At the steady state of the economy, flow creations and flow destructions of every type of agent must exactly balance. For all $(x, y) \in \mathcal{M}$ this implies:

$$\delta h(x, y) = u_m(x) \lambda \frac{u_f(y)}{U_f} = \lambda u_m(x) u_f(y)$$

where $\lambda = \frac{M(U_m, U_f)}{U_m U_f}$ is used to ease exposition. Also notice that $h(x, y) = 0$, $\forall (x, y) \notin \mathcal{M}$. The left-hand side is the flow destruction of existing matches. The right-hand side measures the flow creation as determined by the product of the probability of a contact with the probability of drawing a type $y$ meet. By integrating over $x$ or $y$, this provides the equilibrium amount of $x$-type ($y$-type) match destructions:

$$\delta \int_{(x,y) \in \mathcal{M}} h(x, y) \, dy = \lambda u_m(x) \int_{(x,y) \in \mathcal{M}} u_f(y)$$

$$\delta \int_{(x,y) \in \mathcal{M}} h(x, y) \, dx = \lambda u_f(y) \int_{(x,y) \in \mathcal{M}} u_m(x)$$

(3)

Beyond flows equilibrium, the matching process must also let the economy meet the demographic accounting equations at any point in time. As a result, the whole population of males and females must be distributed over marital status according to:

$$N \int_{(x,y) \in \mathcal{M}} h(x, y) \, dy + u_m(x) = \ell_m(x)$$

$$N \int_{(x,y) \in \mathcal{M}} h(x, y) \, dx + u_f(y) = \ell_f(y)$$

(4)

By substituting the flow of matches defined by (3), this in turn defines the steady-state flow equations at the aggregate level for males and females:

$$\lambda u_m(x) \int_{(x,y) \in \mathcal{M}} u_f(y) \, dy = \delta [\ell_m(x) - u_m(x)]$$

$$\lambda u_f(y) \int_{(x,y) \in \mathcal{M}} u_m(x) \, dx = \delta [\ell_m(x) - u_f(y)]$$

(5)
**Value equations.** Match admissions are driven by the expected surplus from a match. Let $v$ denote the instantaneous payoff of a given match. Then, denoting $r$ the (constant) discount rate, $W_m(v,x)$ the actualized expected flow payoff of a matched $x$-type male and $W_m(v,x)$ the actualized expected flow payoff of an $x$-type single, the value equation for matched individuals is:

$$rW_m(v,x) = v + \delta [W_m(x) - W_m(v,x)] \Leftrightarrow (r + \delta)W_m(v,x) = v + \delta W_m(x).$$

Similarly, an $y$-type female expects: $(r + \delta)W_f(v,y) = v + \delta W_f(y)$. We denote $S_m(v,x) (S_f(v,y))$ the net surplus an $x$-type male (an $y$-type female) derives from the match. By virtue of definition, individual surpluses satisfy:

$$S_m(v,x) = W_m(v,x) - W_m(x) \Leftrightarrow W_m(v,x) = S_m(v,x) + W_m(x)$$

$$S_f(v,y) = W_f(v,y) - W_f(y) \Leftrightarrow W_f(v,y) = S_f(v,y) + W_f(y).$$

Substituting this expressions in the value equations of matched individuals, this defines the individual surpluses from a $v$-match as a function of singles’ values:

$$S_m(v,x) = v - rW_m(x)$$

$$S_f(v,y) = v - rW_f(y)$$

(6)

### 2.3 Instantaneous payoffs

Unlike most existing models of couple formation ([Shimer and Smith, 2000][Atakan, 2006]), we consider an endogenous output from the match. Individual’s payoffs are based on the instantaneous utility $v_m(\ell, c, x, y), v_f(\ell, c, x, y)$ derived from leisure $\ell$ and private consumption $c$ for a male of type $x$ matched to a female of type $y$ (if unmatched, $y = 0$), or for a female $y$ matched with a male $x$ (if single, $x = 0$).

We denote $t$ the monetary transfer between spouses. The transfer shares income between spouses inside formed household, and is sometimes called “sharing rule” in the labor supply literature. For instance, the budgetary constraint of a matched $x$-type man is: $c_m + xl = xT - t$, where $T$ denotes time endowment, while his spouse’s constraint writes: $c_f + yl = yT + t$. We hereafter work with the satisfaction derived from a given match, as defined by the indirect utility functions:

$$\psi_m(x, xT - t; y) = \max_{l<T,c>0} \{v_m(l, c; x, y) | c + xl = xT - t\}$$

$$\psi_f(y, yT + t; x) = \max_{l<T,c>0} \{v_f(l, c; x, y) | c + yl = yT + t\}$$

The indirect utility function should be increasing in total income and decreasing in the own wage ($w = x$ or $y$). For singles, we omit the last component referring to the spouse’s type.
**Singles’ value functions.** Singles’ consumption is entirely founded by their own income. Future prospects involve the opportunity to find an acceptable match, in which instantaneous utility switches to: \( \psi_m (x, xT - t^* (x, y); y) \) and \( \psi_f (y, yT + t^* (x, y); x) \), where \( t^* (x, y) \) is the optimal transfer from husband to wife (possibly negative) that result from the bargaining process inside the household, to be described below. The net surpluses from the match are thus:

\[
S^*_m (x|y) = W_m (\psi_m (x, xT - t^* (x, y); y), x) - W_m (x)
\]

\[
S^*_f (y|x) = W_f (\psi_f (y, yT + t^* (x, y); x), y) - W_f (y)
\]

When a marriage offer arrives, the decision to accept or reject trades the current gain in utility against the future prospect to experience better matches, hence higher surplus. This optimal acceptation rule solves the Bellman equations for singles:

\[
\begin{align*}
\frac{d}{dx} W_m (x) &= \psi_m (x, xT) + \lambda \int S^*_m (x|y) u_f (y) dy \\
\frac{d}{dy} W_f (y) &= \psi_f (y, yT) + \lambda \int S^*_f (y|x) u_m (x) dx
\end{align*}
\]

We now turn to intra-household decisions, from which the surpluses are endogenously derived in our setting.

**Household labor supply.** As became standard in collective models of labor supply, we restrict household choices to the Pareto frontier. Individual decisions result from a Nash bargaining over private consumptions as well as leisures. This reduces to bargain over the transfer \( t^* (x, y) \) that shares income between spouses. Letting \( \beta \) stand for males’ bargaining power, intra-household decisions hence solve:

\[
\max_{t^*(x,y)<x+y} S_m (x|y) \beta S_f (y|x)^{1-\beta}
\]

Thanks to the linearity of \( S_m \) and \( S_f \) in utility, as shown in [6], the FOC of (8) reduces to:

\[
\frac{\beta \partial_t \psi_m [x, xT - t^*(x, y); y]}{\psi_m [x, xT - t^*(x, y); y] - rW_m (x)} = \frac{(1 - \beta) \partial_t \psi_f [y, yT + t^*(x, y); x]}{\psi_f [y, yT + t^*(x, y); x] - rW_f (y)}
\]

This implicitly defines income sharing inside formed household. Spouses’ labour supply and private consumption result from individual decisions along the budget constraint derived from this shared income, through Roy’s identity.

**Equilibrium matching set.** The optimal choices of household members just derived determine the individual optimal surpluses from the match. It is a rational decision to enter an \((x, y)\) match for an \(x\)-type male if the surplus is positive; this match is hence formed iff the surplus from the \((x, y)\) match is positive for a \(y\)-type female. The equilibrium matching set is hence:

\[
\mathcal{M} = \{(x, y) | \exists t^* (x, y) > 0 : S_m (x|y) > 0 \text{ & } S_f (y|x) > 0\}
\]
2.4 Equilibrium

For given preferences, the steady-state equilibrium of the economy is fully-described by the set of functions \( \{ t^*(x, y), M(x, y), u_m(x), u_f(y), rW_m(x), rW_f(y) \} \), i.e. by specifying:

- **Intra-household bargaining**: The transfer \( t^*(x, y) \) optimally shares income between spouses according to (9), given the values of being single \( rW_m(x) \) and \( rW_f(y) \);

- **Who is matched with whom**: \( M(x, y) \) is defined by the optimal surpluses resulting from couple instantaneous decisions, given \( t^*(x, y), u_m(x), u_f(y), rW_m(x) \) and \( rW_f(y) \);

- **How much everyone’s time is worth**: \( rW_m(x) \) and \( rW_f(y) \) solve the implicit system (7), given \( \{ t^*(x, y), M(x, y), u_m(x), u_f(y) \} \);

- **The measure of types searching**: \( u_m(x) \) and \( u_f(y) \) solve the steady-state system (5) given \( M(x, y) \).

By assuming an arbitrarily fixed equal sharing between spouses of an exogenous match output \( f(x, y) \), this equilibrium reduces to Shimer and Smith (2000)'s. We illustrate below how this output is endogenized by specifying an instantaneous utility function.

2.5 An illustration

We illustrate the original contribution of our work by assuming a standard Gorman polar form for indirect utility:

\[
\psi (w, wT + t) = C w^{-\rho} \left[ wT + t - \frac{\alpha}{\rho} - \frac{\gamma w}{1 - \rho} - \frac{\mu w^2}{2 - \rho} \right]
\]

The attractive feature of this specification for our empirical investigations is its flexibility. We add even more flexibility to the model by introducing possible scale effects of matching through the positive function \( C(w) \geq 1 \), assumed an increasing function of spouses’ income, with \( C(0) = 1 \).

Optimal choices according to private leisure and consumption directly follows:

\[
wt^* (w, wT + t) = -w \frac{\partial_w \psi (w, wT + t)}{\partial_t \psi (w, wT + t)}
= \frac{\gamma w^*}{1 - \rho} + \frac{2\mu w^2}{2 - \rho} + \rho \left[ wT + t - \frac{\alpha}{\rho} - \frac{\gamma w}{1 - \rho} - \frac{\mu w^2}{2 - \rho} \right]
= -\alpha + \gamma w + \mu w^2 + \rho (wT + t)
\]
and consumption:

$$c^*(w, wT + t) = wT + t - w^T (w, wT + t) = \frac{\alpha}{\rho} - \frac{\mu w^2}{2 - \rho} + (1 - \rho) \left[ wT + t - \frac{\alpha}{\rho} - \frac{\gamma w}{1 - \rho} - \frac{\mu w^2}{2 - \rho} \right] = \alpha - \gamma w - \mu w^2 + (1 - \rho) (wT + t)$$

Applying (9) to this particular setting, one obtains:

$$t^* (x, y) = (1 - \beta) \left[ C_m (y) \left( xT - \frac{\alpha_m}{\rho_m} - \frac{\gamma_m x}{1 - \rho_m} - \frac{\mu_m x^2}{2 - \rho_m} \right) - rW_m (x) x^{\rho_m} \right] - \beta \left[ C_f (x) \left( yT - \frac{\alpha_f}{\rho_f} - \frac{\gamma_f y}{1 - \rho_f} - \frac{\mu_f y^2}{2 - \rho_f} \right) - rW_f (y) y^{\rho_f} \right]$$

This in turn defines the equilibrium surplus from the match as:

$$S (x, y) = \left[ C_m (y) \left( xT - \frac{\alpha_m}{\rho_m} - \frac{\gamma_m x}{1 - \rho_m} - \frac{\mu_m x^2}{2 - \rho_m} \right) - rW_m (x) x^{\rho_m} \right]$$

$$+ \left[ C_f (x) \left( yT - \frac{\alpha_f}{\rho_f} - \frac{\gamma_f y}{1 - \rho_f} - \frac{\mu_f y^2}{2 - \rho_f} \right) - rW_f (y) y^{\rho_f} \right]$$

An endogenous surplus from the match hence arises from intra-household consumption decisions.

### 2.5.1 Simulating the model

The model is solved through numerical simulations. To do so, we assume some values for the exogenous parameters (to be estimated in the empirical part of the paper) and simulate the resulting equilibrium of the marriage market.

We normalize to one the mass of males and females in the population, and assume a uniform distribution of wages. We allow for gender asymmetries as regards the support of the distribution of wages: $x \in [20, 100]$ while $y \in [10, 50]$. The exogenous parameters driving the matching process are set equal to $\delta = 0.1$, $r = 0.05$.

Intra-household decisions are assumed to be based on an equal bargaining power between spouses: $\beta = 0.5$. We focus on weekly choices by setting the maximum level of leisure, $T$, equal to 100. Individual’s preferences are described by the Gorman specification described in Section 2.5. We account for the effect of matching on individual utility functions by assuming that $b = [\alpha, \gamma, \mu, \rho]$ depends on whether the individual is single, $b_0$, or married ($b_1$). The parameters values used in the simulations are: $b_m = \{[200, 20, 0, 0.5]; [80, 23, 0, 0.5]\}$ for males’ preferences, and $b_f = \{[200, 20, 0, 0.5]; [250, 15, 0, 0.5]\}$ for females’.

Given this values for exogenous parameters, we derive the individual consumptions and optimal surplus using the expressions provided in Section 2.5. We then solve the model by iterating on the
Figure 1: Simulated matching on the marriage market

Notes. Simulated equilibrium of the marriage market based on Gorman polar form utility functions, $T = 100, \delta = 0.1, r = 0.05, \beta = 0.5$ and a uniform distribution over $[20, 100]$ for males’ wage, over $[10, 50]$ for females’ wage. The utility parameters are: $[\alpha, \gamma, \mu, \rho] = [200, 20, 0, 0.5]$ for single males; $= [80, 23, 0, 0.5]$ for married males; $= [200, 20, 0, 0.5]$ for single females and $= [250, 15, 0, 0.5]$ for married females.
equilibrium program provided in 2.4 until convergence to a stable set of equilibrium functions. The steady-state equilibrium of the marriage market is drawn in Figure 1. The most interesting result is the upper-left part of the Figure, displaying the equilibrium matching set: the yellow area is the subspace of wages \((x, y)\) such that \(M(x, y) = 1\). This shows some positive assortative matching on wages can result from labour market decisions, taken as a driving force of matching behavior on the marriage market.

3 Empirical specification

3.1 Data

The empirical identification of the above structural model requires several features. First, our focus on labor market opportunities as a driving force of marriage decisions calls for detailed information on labor market participation. Second, the parameters governing behavior on the marriage market are identified through transitions, \(i.e.\) couple formation. The sampling procedure must therefore be dynamic, in the sense of including the out-of-sample spouse of a newly formed household. Last, beyond the standard demographic characteristics measuring individual traits, a proper accounting of intra-household trade-offs requires to have access to household specific covariates. Child care and support, in particular, are of primary interest.

To all this regards, American data from the *Survey of Income and Program Participation (SIPP)*\(^2\) seems particularly well suited to our framework. Starting from its 1996 wave, the SIPP is a 4-year monthly panel survey including a large representative sample of American people. The aim of the survey is to collect information on households’ well-being, including data on source and amount of income, labor force information, government programs participation and eligibility, and general demographic characteristics.

Interestingly, all members of sampled households (defined as either one person or multi-persons household) are interviewed individually in each wave. The resulting following rule for individuals is that “when original sample members move into households with other individuals not previously in the survey, the new individuals become part of the SIPP sample for as long as they continue to live with an original sample member.” Such a sample design hence provides information on every individual belonging to an observed household, either already existing at the sampling period or recently formed. In addition to the core questions, the survey also includes various specific Topical modules. Topics

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1. Functions are approximated by the image they associate to a given grid of wages’ numerical values. Integrals are approximated by numerical integration techniques, based on Clenshaw-Curtis quadratures.

2. The survey is managed by the US Census bureau and data are freely available; see [http://www.sipp.census.gov/sipp](http://www.sipp.census.gov/sipp) for more information.
Table 1: Distribution of the sample over labour supply and household variables

<table>
<thead>
<tr>
<th></th>
<th>Singles</th>
<th></th>
<th>Married</th>
<th></th>
<th>Gap</th>
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<tr>
<td></td>
<td>Male</td>
<td>Female</td>
<td>Overall</td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>Wages</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>5.00</td>
<td>5.00</td>
<td>5.25</td>
<td>5.00</td>
</tr>
<tr>
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<td>5.60</td>
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</tr>
<tr>
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<td>6.66</td>
<td>6.00</td>
<td>6.25</td>
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<td>6.28</td>
</tr>
<tr>
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<td>7.70</td>
<td>8.05</td>
<td>10.50</td>
<td>8.06</td>
</tr>
<tr>
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<td>11.67</td>
<td>15.00</td>
<td>11.54</td>
</tr>
<tr>
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<td>18.75</td>
<td>16.15</td>
<td>17.30</td>
<td>21.35</td>
<td>16.83</td>
</tr>
<tr>
<td>90% percentile</td>
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<td>24.56</td>
<td>29.85</td>
<td>23.87</td>
</tr>
<tr>
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<td>30.29</td>
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<td>29.57</td>
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<td>18.88</td>
<td>15.48</td>
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</tr>
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<td>10</td>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>5% percentile</td>
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<td>20</td>
<td>22</td>
<td>35</td>
<td>18</td>
</tr>
<tr>
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</tr>
<tr>
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<td>85</td>
<td>90</td>
<td>80</td>
</tr>
<tr>
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<td>40.90</td>
<td>42.39</td>
<td>45.31</td>
<td>38.56</td>
</tr>
<tr>
<td>Standard deviation</td>
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<td>11.69</td>
<td>12.33</td>
<td>11.88</td>
<td>11.69</td>
</tr>
<tr>
<td>Number of observations</td>
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<td>168 928</td>
<td>285 516</td>
<td>231 865</td>
<td>231 865</td>
</tr>
<tr>
<td>Frequency over all observations</td>
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<td>22.55</td>
<td>38.11</td>
<td>30.95</td>
<td>30.95</td>
</tr>
<tr>
<td>Average number of children</td>
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<td>0.65</td>
<td>0.47</td>
<td>1.06</td>
<td>1.06</td>
</tr>
</tbody>
</table>

Notes. Descriptive statistics on individuals included in the sample. The upper part provides wages distributions. Each cell reports the hourly wage in USD producing the point of the empirical pdf that appears in row, conditional on the individual being: a (male or female) single (first three columns), or (male or female) spouse of a married household (last three columns). The last column reports the within household gap, i.e. the distribution of the difference between males’ and females’ wage. The middle part reports the same information as regards hourly hours of work. The last three rows report the number of observations in the sample, the distribution of the sample over sex and marital status and the average number of children below 18 years old.
covered by the modules include personal history, child care, wealth, child support, utilization and cost of health care, disability, school enrollment, taxes, and annual income.

The survey is conducted each month based on a four-group rotation scheme. The 1996 wave starts in December 1995, including only the first rotation group. The second rotation group is introduced in January 1996, hence answering the survey (for the first time) at this date along with the first rotation group. The third rotation group is then introduced in February 1996, and so on. March 1996 is the first date providing information on all individuals included in the panel. Each rotation group is questioned 48 times. The last observation on individuals from the first rotation group is November 1999; December 1999 for the second rotation group, and so on. We work on a balanced panel by reducing the observation window to March 1996-November 1999.

The SIPP reports information on every individual belonging to a sampled household (defined as same roof peoples). This includes every relative or friend of the reference person (i.e. the one who is originally sampled). Our focus is on spouses inside – possibly one-person – households. We hence consider the reference person as the “main” unit of observation, keeping track only of (i) his own labor market and demographic characteristics and (ii) matching choices on the marriage market. As a result, we hold information on new comers to the sample due to marriage matching with a reference person. This choice leads us to disregard the individual matching and labor market behavior of every person who is linked with the reference person by any relation other than marriage. As an example, a child who originally lives with his parents, and leaves to a new address after marriage during the sample period will not be considered in the analysis.

Table 1 provides descriptive statistics on individuals included in the sample over the whole period. Married spouses “produce” on average 1 children, which is one half more than singles. As expected, empirical distributions of wages and hours of work are quite different as regards both sex and marital status. The distributions for married males and females stochastically dominate at first-order their counterpart for singles. Conditional on marital status, male’s distributions dominate at first order females’ ones. Interestingly, those conditional distributions hide non monotonic gaps between spouses within households: the wage gap becomes positive only after the first quarter of the wage distribution of married males; hours gap becomes positive after the first ten percent of married males’ hours distribution.

This last feature reflects the endogenous matching on the marriage market. Figure 2 provides an approximation of the observed matching on the marriage market, based on 95% confidence intervals of females’ wage conditional on males’ one on the left-hand side, of females’ hours of work conditionnal on males’ hours on the right-hand side. Figure 2a is an approximation of the empirical matching set. The observed marriage market gives rise to positive assortative matching on spouses’ wage. The upper bound on the wage of accepted match is increasing in own wage. The picture is less convincing, however, as regards the lower bound on accepted match, which is only slightly increasing in males’
wage. Figure 2b provides a picture of within household bargaining as regards hours of work. The pattern is more or less the same as what happens on the wage side of the marriage market: the upper bound on hours worked by females spouses is increasing in males’ hours, while the lower bound is roughly flat.

The resulting conditional wage distributions of singles are provided in Figure 3. For both males (Figure 3a) and females (Figure 3b), most of the weight is on the bottom of the wage distribution.

3.2 Econometric model

3.2.1 Parameters of interest

Our structural approach seeks to estimate the exogenous parameters driving matching decisions and labour supply behavior. On the one hand, as detailed above (Section 2), the marriage market is fully described by the destruction rate $\delta$, the individual discount rate $r$, the bargaining power of males $\beta$ and the matching function $M$. As standard in the search literature, we assume a Cobb-Douglas functional form with constant returns to scale: $M(U_m, U_f) = A U_m^{\phi} U_f^{1-\phi}$, hence defined by two parameters, $\alpha$ and $A$. 

Legend. Red curve: Upper bound of accepted wages (hours); Blue curve: Lower bound of accepted wages (hours).

Notes. 95% confidence intervals on accepted matches in the sample. The figures are built using the 50 percentiles discrete values on the conditional pdf of married males’ labour market variable, wages on the left-hand side, hours on the right hand side. At each point of the males’ wage (hours) distribution, we compute the 5th and 95th percentiles of spouses wage (hours) distribution. The first one approximates the lower bound of accepted match, the second one provides the upper bound of accepted match. To ease graphical representation of the matching set, the first and last 2%-centiles of married individuals are omitted in the Figures.
3.2.2 Likelihood function

Given some stochastic specifications, the model we developed in Section 2 fully describes the probability distribution of individual trajectories on both the marriage and the labour markets. We hence build the sample likelihood function along to the following dimensions:

- The initial marital status at first sampling date;
- The individual transitions on the marriage market.
- The individual choices on the labour market at each sample date conditional on marital status and observed wage.

**Initial marital status.** For a given observed wage, \( w_i \), the probability the individual is observed single is \( \frac{u_g(w_i)}{U_g} \); the probability of an existing match with a \( w_{-i} \)-type partner is \( \frac{h(w_i, w_{-i})}{L_g} \). Denoting \( s_i = 0 \) a married individual in \( t = 0 \) (\( s_i = 1 \) for singles), the individual contribution to the likelihood
from the initially observed marital status is then $ll_{status} = (ll^{m}_{status})^{g_{i}=m} \times (ll^{f}_{status})^{g_{i}=f}$, with:

$$ll^{m}_{status} = \left( \frac{u_{m}(x_{i})}{U_{m}} \right)^{s_{i}} \left( \frac{h(x_{i},y_{i})}{L_{m}} \right)^{1-s_{i}} \quad \text{for males (} g_{i}=m \text{)}$$

$$ll^{f}_{status} = \left( \frac{u_{f}(y_{i})}{U_{f}} \right)^{s_{i}} \left( \frac{h(x_{i},y_{i})}{L_{f}} \right)^{1-s_{i}} \quad \text{for females (} g_{i}=f \text{)}$$

**Transitions on the marriage market.** We define 4 regimes depending on whether the marital status changes or not, for both singles and married individuals.

**R₁ (Ongoing celibacy)** An individual remains single if either no offer is received or the received offer is outside the matching set:

$$ll^{m}_{R₁} = (1 - \lambda_{m}) + \lambda \int_{y \notin M(x_{i})} u_{f}(y)dy \quad \text{for males (} g_{i}=m \text{)}$$

$$ll^{f}_{R₁} = (1 - \lambda_{f}) + \lambda \int_{x \notin M(y_{i})} u_{m}(x)dx \quad \text{for females (} g_{i}=f \text{)}$$

**R₂ (Wedding)** A single becomes married by accepting a received offer. The probability of this event is:

$$ll^{m}_{R₂} = \lambda u_{f}(y_{i}) \quad \text{for males (} g_{i}=m \text{)}$$

$$ll^{f}_{R₂} = \lambda u_{m}(x_{i}) \quad \text{for females (} g_{i}=f \text{)}$$

**R₃ (Ongoing marriage)** Couples remain the same as long as no exogenous destruction occurs:

$$ll^{m}_{R₃} = ll^{f}_{R₃} = (1 - \delta)$$

**R₄ (Divorce)** Previously married individuals get back to the pool of singles when destruction occurs:

$$ll^{m}_{R₄} = ll^{f}_{R₄} = \delta$$

Defining $R_{k,i}$ as an index denoting the observed transition in date $t$ (index omitted for clarity), the individual contribution to likelihood from observed transition in $t$ writes:

$$ll_{transition} = \left( \prod_{k=1}^{4} (ll^{m}_{R_{k}})^{R_{k,i}} \right)^{g_{i}=m} \times \left( \prod_{k=1}^{4} (ll^{f}_{R_{k}})^{R_{k,i}} \right)^{g_{i}=f}$$
Labour supply. The probability associated with the number of hours worked at each date is conditional on current wage and marital status. We derive a stochastic specification from the functional form described in Section 2.5 by assuming an i.i.d., additively separable and log-normally distributed measurement error:

\[ l^m_i(x_i, y_i) = \frac{1}{x_i}(-\alpha^m_i + \gamma^m_i x_i + \mu^m_i x_i^2 + \rho^m_i (x_i T + t(x_i, y_i))) + \varepsilon^m_i \text{ for married males } (g_i = m, s_i = 1) \]

\[ l^m_i(x_i, 0) = \frac{1}{x_i}(-\alpha^m_0 + \gamma^m_i x_i + \mu^m_0 x_i^2 + \rho^m_0 x_i T) + \varepsilon^m_i \text{ for single males } (g_i = m, s_i = 0) \]

\[ l^f_i(x_i, y_i) = \frac{1}{y_i}(-\alpha^f_i + \gamma^f_i y_i + \mu^f_i y_i^2 + \rho^f_i (y_i T - t(x_i, y_i))) + \varepsilon^f_i \text{ for married females } (g_i = f, s_i = 1) \]

\[ l^f_i(0, y_i) = \frac{1}{y_i}(-\alpha^f_0 + \gamma^f_0 y_i + \mu^f_0 y_i^2 + \rho^f_0 y_i T) + \varepsilon^f_i \text{ for single females } (g_i = f, s_i = 0) \]

The income sharing is defined as: \( t(x_i, y_i) = (1 - \beta)S_m(x_i) + (1 - \beta)S_f(y_i) \) and \( t(x_i, 0) = t(0, y_i) = 0 \). Denoting \( f(.) \) the pdf of the standardized log-normal distribution, the probability of observing \( \hat{l}_i \) hours at any date \( t \) is given by: \( f[\hat{l}_i - l^m_i(x_i, y_i)] \). Remember we observe both spouses’ hours of work would a sample individual be married. Since the measurement errors are assumed independent, the joint probability of hours worked for couples simply reduces to the product of the probability for each spouse. Denoting \( j_i \) the outside sample spouse of individual \( i \), this leads to:

\[ l_{\text{hours}} = \left[f[\hat{l}_i - l^{m_i}(x_i, y_i)]\right]^{s_i} \times \left[f[\hat{l}_i - l^{f_i}(x_i, y_i)] \times f[\hat{l}_{j_i} - l^{f_{j_i}}(x_i, y_i)]\right]^{1-s_i} \]

The individual contribution of individual \( i \) for all sample dates is then:

\[ l_{\text{i}} = l_{\text{status}} \times \prod_t l_{\text{transition}}^t \times \prod_t l_{\text{hours}}^t \]

### 3.2.3 Estimation

Note that the computation of the above individual contribution requires an evaluation of the stationary equilibrium of the model (which defines, e.g., the density of singles that appear in the contribution from marital status). We hence mix the likelihood maximization procedure with a simulation step: at each iteration of the model, we use the simulation procedure described in Section 2.5.1 to derive the equilibrium of the model at current parameters value. We then compute the likelihood function, update the parameters using an optimization algorithm and repeatedly get back to simulation and likelihood computation until convergence.
References


