Collective Model with Children: Public Good and Household Production

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Abstract

The present paper develops a theoretical model of labor supply with domestic production and public goods. The objective of the paper is to deal simultaneously with these two aspects and thus consider a good that is both produced within the household and publicly consumed by household members. In the present work, this good is represented by the quality and quantity of household children. In particular, we are interested in the cost of children and in the way this cost is shared between parents. The total cost of children is made both of time and money, in the sense that it is defined as a market consumption cost plus the remuneration of parental time devoted to take care of children. These theoretical considerations are followed by an empirical application using French data (EDT).

Key-words: colletive model, market labor supply, domestic labor supply, household production, identification, children cost, public good, EDT

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1 Introduction

This paper aims at modeling household decisions concerning children in a collective approach. The original framework, initially developed by Chiappori [8], considers agents who individually maximize an egotistic utility function, defined over their private consumption of market goods and leisure, subject to the Pareto constraints that the partner's utility is not less than a given level of welfare, the household budget constraint, and individual time constraints. Alternatively, the household maximizes a weighted sum of individual utilities with respect to the household budget constraint and individual time constraints. Chiappori [9] shows that an alternative interpretation is given by the existence of two stages in the household internal decision process: household members first share non labor income, according to a given sharing rule, and then each one chooses his or her own labor supply and consumption. Finally, Chiappori [9] shows that the model still holds in the case of caring (not paternalistic) preferences.

Browning et al. [7] have generalized the collective approach by introducing public goods. Another generalization (Apps and Rees [1] and Chiappori [10]) of the collective approach considers household production of a (marketable or non marketable) good consumed privately by household members. The objective of our paper is to deal simultaneously with these two aspects and thus consider a good that is both produced within the household and publicly consumed by household members. In our paper, this good is represented by the quality and quantity of household children. However, this seems to us the most suitable way of modeling household decisions concerning children.

In particular, we are interested in the cost of children and in the way this cost is shared between parents. The total cost of children is made both of time and money (Apps and Rees [2], Bradbury [6], and Perali et al. [15]) in the sense that it is defined as a market consumption cost plus the remuneration of parental time devoted to take care of children. Although children are modeled in a similar way in other works (Blundell et al. [3], Donni [13]), to the best of our knowledge, the way we propose to model how the cost of children is shared among spouses has never been analyzed up to now.

Under weak assumptions concerning the technology of production for the public good (either constant or decreasing returns to scale; quasi concavity), combined with the usual assumptions concerning preferences (individual utilities strictly increasing and quai concave in their arguments; separability in individual utilities betweeen the public good and the private sphere that involves consumption and leisure), we recover the cost of children borne by each parent.

More precisely, assuming stable preferences of household members, the individual contribution to the total cost of children is given by the difference between the amount that an household member (in our case, the husband or the wife) would get if there are not household children and the amount that the same individual gets once there are household children. Or alternatively, the individual contribution to the total cost of children is given by the difference between the amount that an household member would get if spouses share household non labor income and the amount that the same individual gets once spouses share the household exogenous income minus the total production cost. In other words, how much the husband (or the wife) gets if he and his (her) partner share household non labor income? And, how much the husband (or the wife) gets if he and his (her) partner share household non labor income minus the total production cost? The difference between these two amounts gives the the husband (or the wife) constribution to the total production cost. Namely, the individual contribution to the total production cost is given by the difference between the sharing rule defined over exogenous household income and the sharing rule defined over the exogenous household income minus total production cost.

The paper is organised as follows. In section 2, we present two equivalent pareto programs that allows us to define the individual contribution to the total production cost as difference between the two sharin rule previously mentioned. In section 3, we present identification results of the sharing rule and the individual contribution to the total production cost. In section 4 we discuss the construction of the index that explains the quality and quantity of household children. In sections 5 and 6, we present, respectively, empirical specification and data sources. Finally, in section 7, we discuss estimation results.

2 The Pareto-Optimal Case

2.1 Definitions and program

In what follows, we consider a two-person household which members $i = 1$, *2*, respectively the husband and the wife, are characterized by his or her own rational preferences. Index *3* denotes what is outside the household. All the analysis is conducted under the hypothesis that each spouse is selfish toward the other and that, whatever the decision making process inside the household, the spouses will always exploit all consumption opportunities and they will come up to a Pareto efficient allocation.

Household members work for money (t^1 and t^2 hours paid at the wage rate w_1 and w_2), enjoy leisure l^1 and l^2, consume a private good c^1 and c^2 (what we empirically observe is $C = c^1 + c^2$) and a quantity of a public good y that represents both the quantity and quality of household children. The price of the private composite good is normalized to 1.

As shown by Chiappori and Ekeland [11], when there is a public good, identification can be achieved only under the hypothesis of separability in the individual utilities between the public good y and the private sphere that involves consumption c^i and leisure l^i.

Assumption 1. *Individual utilities are characterized by egoistic preferences of the form:*

$$U^i\left(u^i\left(c^i, l^i; Z_p\right), y; Z_p\right)$$

where Z_p denotes the vector of individual characteristics that affect preferences and U^i and u^i are strictly increasing in their arguments and strictly quasi concave and verify the Inada conditions (except that the marginal utility of y is not infinite when $y = 0$).

In this model, y is a good that is both publicly consumed by household members (in the sense that the husband and the wife enjoy the same quantity of the public good) and produced within the household. In fact, each member shares his time T between leisure, market work and domestic work h^i, so that:

$$T = l^i + t^i + h^i$$

where h^i represents the time spent by each parent to take care for their children. In order to produce y, the household also buys some input goods and services (such as clothing, school insurance, school meals, transport, education, etc.), which values is denoted as c^3, and time inputs h^3 such as a nurse, a baby-sitter, etc.) paid at the exogenous wage w_3. Given the available information, we must suppose that w_3 does not change among household. Then, let define the household monetary cost of producting y as $c^y \equiv w_3 h^3 + c^3$. Notice that input goods and time inputs are evaluated at their market price, and therefore valuated the same way by the husband and the wife. Then, the household total cost of producing the public good is

$$TC = w_1 h^1 + w_2 h^2 + c^y.$$

The household production technology is given by [1]:

$$y = Y\left(h^1, h^2, c^y; Z_h\right)$$

Assumption 2. *The function Y is increasing and concave in each argument and globally quasi-concave. Z_h denotes the vector of household characteristics that affect production decisions.*

Notice that we only make the assumption of decreasing (or constant) marginal productivities for each input but, unlike Chiappori [10] and Apps and Rees [1], we need not to assume constant returns to scale. In what follows, we consider the case of some complementarity between inputs.

Finally, household non labor income is denoted by m.

Any Pareto-Optimal solution solves the following constrained maximization program (P0), namely it maximizes a linear social welfare function

$$Max\ \lambda U^1\left(u^1(c^1, l^1), y\right) + (1-\lambda)U^2\left(u^2(c^2, l^2), y\right)$$

subject to the household budget constraint, the household production technology and individual time constraints:

$$c^1 + c^2 + c^y + w_1 l^1 + w_2 l^2 + w_1 h^1 + w_2 h^2 \leq w_1 T + w_2 T + m$$

$$y \leq Y(h^1, h^2, c^y)$$

$$l^i + t^i + h^i \leq T, \quad i = 1, 2.$$

The function λ represents the Pareto-weight that depends on the exogenous variables entering the budget constraint, such as wages and non labor income, and on distributional factors s, that are variables that influence the decision process without affecting the budget set or preferences. An interpretation of this welfare weight is that it represent the *bargaining power* of the individual

[1]It corresponds to a more complete production function $\widetilde{Y}\left(h^1, h^2, h^3, c^3; Z_h\right))$

1 in the intra-household allocation process. Namely, λ determines the final position on the Pareto-frontier. Changes in wages or non labor income may shift bargaining power from one individual to the other, with consequences on observable household consumption and labor supply.

2.2 The two-stages program

Both egoistic preferences and caring preferences make an alternative resolution of the program P0 possible. In fact, an application of the second fundamental theorem of welfare economics allows us to solve the program in different stages.

At the first stage of the program P1, the production level of the public good y is collectively chosen, and technically, it is a fuction of all exogenous variables (w_1, w_2, m, s, Z_h, and Z_p). But, just as the value the level of expenditure for the publig good (K) in Blundell et al. [3], in what follows, $y = y^*(w_1, w_2, m, s, Z_h, Z_p)$ is taken as given (otherwise it is predetermined and for the enveloppe theorem we don't have to derive y with respect to exogenous variables because it is the optimum).

The minimization of household total cost of producing the domestic good, given the production function of the public good, gives optimal inputs levels. Optimality and interior solutions [2] imply following first order conditions[3]:

$$\frac{Y_{h^i}}{Y_{c^y}} = w_i, \quad i = 1, 2. \tag{1}$$

which gives unique inputs levels of the individual domestic labor supply $h^i = h^i\,(y, w_1, w_2; Z_h)$ and of the monetary cost $c^y = c^y(y, w_1, w_2; Z_h)$. This relation says that for individual i the marginal value of time spent in household production, namely h^i, relative to the monetary cost c^y is equal to his

[2]For corner solutions, see Blundell et alii [4] and Donni [14]

[3]In what follows, the notation X_k stands for the partial differential of the function X with respect to variable k.

market wage. In other words, household member i is marginally indifferent between one hour spent in household production and w_i\$ spent for c^y. Given the optimal input levels, the total cost of producing the public good is defined as:

$$TC(y, w_1, w_2; Z_h) = w_1 h^1(y, w_1, w_2; Z_h) + w_2 h^2(y, w_1, w_2; Z_h) + c^y(y, w_1, w_2; Z_h).$$

At this point, household income non labor income net of the cost of producing the public good $m - TC(y, w_1, w_2; Z_h)$ is shared between spouses, according to a sharing rule Ψ:

$$\Psi^1 = \tilde{\Psi}(m, w_1, w_2, y, h^1(.), h^2(.), c^y(.), s; Z_h, Z_p) = \Psi(m, w_1, w_2, y, s; Z_h, Z_p)$$

and, omitting the explanatory variables,

$$\Psi^2 = m - TC - \Psi^1.$$

The existence of a sharing rule implies no more (and not less) than the efficiency of the collective decision process (see Chiappori [9]). Given any λ we can find a sharing rule Ψ such that the outcomes of the two associated programs are the same and vice versa, in other words there is a one to one correspondence between λ and Ψ. This means that bargaining power within the household can be measured alternatively by any of those functions since they are equivalent. The sharing rule depends both on the level of y and also on the level of each input. Pareto-optimal decisions taken by spouses at the first stage can be seen as individually optimal in the sense that, because individual i anticipates the impact of her decisions on the sharing rule, she has no incentive to deviate from the Pareto-optimal solution.

At the second stage, each spouse, separately, chooses how to allocate her own budget between composite private consumption good and leisure. In other words, each household member maximizes her own utility

$$u^i\left(c^i, l^i; Z_p\right)$$

under her own budget constraint implied by previous steps, namely

$$c^i + w_i l^i \leq w_i T + \Psi^i.$$

Optimality and interior solutions imply following first order conditions:

$$\frac{u_l^i}{u_c^i} = w_i, \quad i = 1, 2. \tag{2}$$

This relation represents the marginal value of leisure relative to the private consumption good. For the individual i, one hour spent enjoying leisure is marginally equivalent to w_i\$ spent for the private consumption good c^i. Demand functions for consumption are $c^i = c^i\left(w_i, \Psi^i\left(w_1, w_2, m, y, s; Z_h, Z_p\right); Z_p\right)$. Demand functions for leisure are $l^i = l^i\left(w_i, \Psi^i(w_1, w_2, m, y, s; Z_h, Z_p\right); Z_p)$. Moreover, from the time constraints, we can determine the Marshallian total labor supply funtion

$$L^i = T - l^i = t^i + h^i = L^i\left(w_i, \Psi^i\left(w_1, w_2, m, y, s; Z_h, Z_p\right); Z_p\right).$$

Finally, the associated individual indirect utility functions are:

$$v^i(w_i, \Psi^i) = u^i\left(c^i\left(w_i, \Psi^i; Z_p), l^i(w_i, \Psi^i; Z_p\right); Z_p\right). \tag{3}$$

This approach does not allow to determine explicitly how much each spouse contributes to the monetary cost c^y of the public good, even if the first two stages implicitly define such a repartition. Our goal is to make it explicit, then, in the next section, we develop an alternative way of presenting the traditional approach.

2.3 How total production cost is shared between parents?

In this section, we present a second program, named P2, totally equivalent to program P1, but with two advantages. First, it makes explicit the implicit (because not actually paid) repartition of the monetary cost c^y between spouses, namely c^{y1} and c^{y2}; second, it allows to define and measure the individual total cost of children borne by each parent. As we said in section 2.1, the household total cost of producing the public good is made by a monetary cost c^y and the remuneration of parents' time devoted to children, namely $TC = w_1 h^1 + w_2 h^2 + c^y$. Our goal is to measure how much of that cost is implicitly borne individually by each parent. In other words, we want to determine $TC^1 = w_1 h^1 + c^{y1}$ and $TC^2 = w_2 h^2 + c^{y2}$, where $c^{y1} = \alpha C_y$ and $c^{y2} = (1 - \alpha) C_y$.

The knowledge of time devoted by each spouse to take care for children, namely the domestic work h^i in the production function, cannot give any idea of how much each parent implicitly spends for children because children's cost may be compensated in the sharing rule. Let us consider the following example. Suppose that in a family the father cares more for children than the mother (namely, he is willing to spend more than the mother for child care) but he is less productive in household production. This means that the wife will spend a large amount of time with children but she will be compensated by the husband through the sharing rule. This allows her to increase her consumption of the market private good and leisure. At the same time, the husband cares more for children and then he undergoes a greater share of the children cost c^y that reduces his share of the household income. On the other hand, if the mother spends a large amount of time with children because she cares a lot for them (more than husband), this will not be compensated in the sharing rule. This means that she will have a lower share of household income that will force her to reduce her consumption of private composite good and leisure.

Then, let us consider the following two-stages program, named P2. At the first stage, household members agree on the repartition of family non labor income according to a sharing rule Φ, as if they still had to contribute to the production of the public good. Then, let us define

$$\Phi^1 = \Phi(w_1, w_2, m, s; Z_p)$$

and, omitting the explanatory variables, $\Phi^2 = m - \Phi$.

Notice that Φ is not affected by the price of market time w_3, because household production decisions are taken in a following step. Once defined the sharing rule Φ, in fact, the production level of y is collectively chosen (just as in the first stage of program P1). Then the minimization of household cost of producing the domestic good gives optimal inputs levels $h^i = h^i(y, w_1, w_2; Z_h)$ and $c^y = c^y(y, w_1, w_2; Z_h)$. At this point, the household monetary cost for children c^y is shared between spouses according to a sharing rule α, which gives $c^{yi} = \alpha_i c^y$ with $\alpha_1 + \alpha_2 = 1$.

For given levels or y, c^y, h^1 and h^2, the repartition of monetary cost affects spouses' utilities, but not the production sphere because of the separability. The optimal repartition α should then maximize:

$$\lambda U^1\left(v^1(w_1, \Phi^1 - w_1 h^1 - \alpha c^y), y\right) + (1-\lambda) U^2\left(v^2(w_2, \Phi^2 - w_2 h^2 - (1-\alpha) c^y), y\right)$$

which gives the first order condition:

$$\lambda U_u^1 v^{1\prime} = (1-\lambda) U_u^2 v^{2\prime}. \qquad (4)$$

We obtain that the marginal utilities of income are inversely proportional to the Pareto-weight. This first order condition is sufficient, so there exists a unique solution $\alpha \in \Re$, where:

$$\alpha = \tilde{\alpha}(m, w_1, w_2, y, h^1(.), h^2(.), c^y(.), \Phi^1(.), \Phi^2(.), Z_p, Z_h)$$

There is no reason why α should lie in the interval [0,1]. Indeed, it may well be the case that $\alpha^1 = \alpha < 0$ (or $\alpha^2 = (1-\alpha) < 0$), for example if individual *1* (or individual *2*) is very productive in taking care of children. In that case, individual *1* (or individual *2*) will spend a lot of time in household production, but she will be compensated reducing her contribution to the monetary cost (just to have a negative c^{yi} because α^i is negative). In any case, even if the individual monetary cost may be negative for one member, the individual total cost $TC^i = w_i h^i + \alpha^i c^y$ (aggregating remuneration of household working time and individual monetary cost) is always positive for both spouses. This means that, at the second stage, both spouses will face a lower income: $\Phi^1 - w_1 h^1 - \alpha c^y < \Phi^1$ and $\Phi^2 - w_2 h^2 - (1-\alpha) c^y < \Phi^2$. Since $TC^i > 0$, this implies that:

$$-\frac{w_1 h^1}{c^y} < \alpha < 1 + \frac{w_2 h^2}{c^y}$$

Finally, at the second stage, each spouse separately maximizes her own utility

$$u^i\left(c^i, l^i; Z_p\right)$$

under her own budget constraint implied by previous steps, namely

$$c^i + w_i l^i \leq w_i T + \Phi^i - TC^i.$$

Demand functions for consumption and leisure are $c^i = C^i(w_i, \Phi^i - TC^i; Z_p)$ and $l^i = l^i(w_i, \Phi^i - TC^i; Z_p)$. Moreover, from the individual time constraint, we can determine the Marshallian total labor supply funtion

$$L^i = T - l^i = t^i + h^i = L^i(w_i, \Phi^i - TC^i; Z_p).$$

Notice that both α_i and the income left to each individual's private consumption $\Phi^i - TC^i$, are affected by the level of y and by the level of each input h^i and c^y.Then, the Pareto-optimal decisions taken by household at the first stage can be seen as individually optimal because, since spouse i anticipates the impact of her decision on the following stage, she has no incentive to deviate from the Pareto-optimal solution.

To conclude, let us remark that the equivalence between the programs P1 and P2 implies that the net income available to each spouse after she has contribute to household production, that is $\Phi^i - TC^i$ is equal to the sharing rule Ψ^i defined in program P1. This allows us to determine the value of individual total cost as difference between the sharing rule defined over the non labor income and the sharing rule defined over the exogenous income minus total production cost, that is

$$TC^i(w_1, w_2, y, m, s; Z_p, Z_h) = \Phi^i(w_1, w_2, m, s; Z_p) - \Psi^i(w_1, w_2, y, m, s; Z_p, Z_h).$$

More precisely, assuming stable preferences of household members, the individual contribution to the total cost of children is given by the difference between the amount that an household member (in our case, the husband or the wife) would get if there are not household children and the amount that the same individual gets once there are household children. Or alternatively, the individual contribution to the total cost of children is given by the difference between the amount that an household member would get if spouses share household non labor income and the amount that the same individual gets once spouses share the household exogenous income minus the total production cost. In other words, how much the husband (or the wife) gets if he and his (her) partner share household non labor income? And, how much the husband (or the wife) gets if he and his (her) partner share household non labor income minus the total production cost? The difference between these two amounts gives the the husband (or the wife) constribution to the total production cost.

2.4 How is the efficiency condition for the public good?

In what follows, we show that the efficiency condition for the public good takes the standard Bowen-Lindhal-Samuelson form. Given the production function

$$y = Y[h^1(y, w_1, w_2), h^2(y, w_1, w_2), c^y(y, w_1, w_2)]$$

deriving it with respect to y we have:

$$1 = \frac{\partial Y}{\partial h^1}\frac{\partial h^1}{\partial y} + \frac{\partial Y}{\partial h^2}\frac{\partial h^2}{\partial y} + \frac{\partial Y}{\partial c^y}\frac{\partial c^y}{\partial y}$$

and substituting relation (1), we obtain:

$$\frac{1}{Y_{c^y}} = w_1 h^1_y + w_2 h^2_y + c^y{}_y. \tag{5}$$

At the first stage of program P2, Pareto-optimality implies that c^y, h^1 and h^2 maximize the household welfare function:

$$\lambda U^1\left(v^1(w_1, \Phi^1 - w_1 h^1 - \alpha c^y), Y(h^1, h^2, c^y)\right)$$
$$+(1-\lambda)U^2\left(v^2(w_2, \Phi^2 - w_2 h^2 - (1-\alpha)c^y), Y(h^1, h^2, c^y)\right)$$

which gives first order condition for c^y:

$$\lambda U^1_u v^{1\prime}(-\alpha) + \lambda U^1_y Y_{c^y} + (1-\lambda)U^2_u v^{2\prime}(\alpha - 1) + (1-\lambda)U^2_y Y_{c^y} = 0$$

where $(1-\lambda)U^2_u v^{2\prime} = \lambda U^1_u v^{1\prime}$ because of (4)[4]. Then, we obtain:

$$\frac{1}{Y_{c^y}} = \frac{U^1_y}{U^1_u v^{1\prime}} + \frac{U^2_y}{U^2_u v^{2\prime}}. \tag{6}$$

Finally, combining equations (6) and (5), we obtain that the optimal level of y is such that the marginal cost of producing y for the household is equals to

[4]Similarly, for h^1 and h^2. The first order condition, respectively, for h^1 and h^2 are

$$\lambda U^1_u v^{1\prime}(-w_1) + \lambda U^1_y Y_{h^1} + (1-\lambda)U^2_y Y_{h^1} = 0$$

$$\lambda U^1_y Y_{h^2} + (1-\lambda)U^2_u v^{2\prime}(-w_2) + (1-\lambda)U^2_y Y_{h^2} = 0$$

Then, rearrangin, we obtain:

$$\frac{w_i}{Y_{h^i}} = \frac{U^1_y}{U^1_u v^{1\prime}} + \frac{U^2_y}{U^2_u v^{2\prime}} = \frac{1}{Y_{c^y}}.$$

We find again that the marginal value of h_i relative to c^y is equal to w_i, as expected if labor market is competitive.

the sum of the marginal benefits of consuming y for the two spouses, other ways to the sum of the marginal amounts spouses are eager to pay in order to enjoy one unit more of y. Let us define:

$$p_i \equiv \frac{U_y^i}{U_u^i v^{i\prime}} \tag{7}$$

the marginal value of y for individual i, that is i is marginally indifferent between one unit more of y and p_i dollars to share between her private consumption and leisure. The p_i's simply correspond to Lindhal prices.

Then, we can show that, at the optimum, the relative marginal value for y for the spouses only depends on their own relative preferences for the public good and on Pareto-weight, but not on the production side:

$$\frac{p_2}{p_1} = \frac{U_y^2}{U_u^2 v^{2\prime}} \frac{U_u^1 v^{1\prime}}{U_y^1} = \frac{(1-\lambda)}{\lambda} \frac{U_y^2}{U_y^1}$$

where the last equality derives from relation (4).

3 Identification results

In following sections we show identification results concerning the sharing rule $\Psi(w_1, w_2, y, m, s; Z_p, Z_h)$ and the individual total cost $TC^i(w_1, w_2, y, m, s; Z_p, Z_h)$.

3.1 Restrictions on total labor supplies and the sharing rule

Chiappori [12] shows that a set of testable restrictions of the collective approach on observable market labor supplies can be derived and that the sharing rule $\Phi(w_1, w_2, m, s; Z_p)$ can be identified up to an additive constant.[5]

[5] Chiappori [12] identifies the partial derivatives of the sharing rule Φ in terms of observable labor supplies t^1 and t^2, namely: $\Phi_{w_2} = \frac{AD}{D-C}$, $\Phi_{w_1} = \frac{BC}{D-C}$, $\Phi_s = \frac{CD}{D-C}$, $\Phi_m = \frac{D}{D-C}$ where: $A = t^1_{w_2}/t^1_m$, $B = t^2_{w_1}/t^2_m$, $C = t^1_s/t^1_m$, and $D = t^2_s/t^2_m$.

In this section we show that a set of testable restrictions of the collective approach on observable total labor supplies can be derived and that the sharing rule $\Psi(w_1, w_2, y, m, s; Z_p, Z_h)$ can be identified up to an additive constant. Then, following Chiappori, we differentiate total labor supply functions

$$L^1 = t^1 + h^1 = L^1\left(w_1, \Psi^1(w_1, w_2, y, m, s; Z_p, Z_h); Z_p\right)$$
$$L^2 = t^2 + h^2 = L^2\left(w_2, \Psi^2(w_1, w_2, y, m, s; Z_p, Z_h); Z_p\right)$$

with respect to wages, non labor income, the level of public good, and the distribution factor:

$$\frac{\partial L^1}{\partial w_2} = \frac{\partial L^1}{\partial \Psi^1}\frac{\partial \Psi^1}{\partial w_2}$$
$$\frac{\partial L^1}{\partial s} = \frac{\partial L^1}{\partial \Psi^1}\frac{\partial \Psi^1}{\partial s}$$
$$\frac{\partial L^1}{\partial y} = \frac{\partial L^1}{\partial \Psi^1}\frac{\partial \Psi^1}{\partial y}$$
$$\frac{\partial L^1}{\partial m} = \frac{\partial L^1}{\partial \Psi^1}\frac{\partial \Psi^1}{\partial m}$$
$$\frac{\partial L^2}{\partial w_1} = \frac{\partial L^2}{\Psi^2}\left(-\frac{\partial TC}{\partial w_1} - \frac{\partial \Psi^1}{\partial w_1}\right)$$
$$\frac{\partial L^2}{\partial s} = \frac{\partial L^2}{\partial \Psi^2}\left(-\frac{\partial \Psi^1}{\partial s}\right)$$
$$\frac{\partial L^2}{\partial y} = \frac{\partial L^2}{\partial \Psi^2}\left(-\frac{\partial TC}{\partial y} - \frac{\partial \Psi^1}{\partial y}\right)$$
$$\frac{\partial L^2}{\partial m} = \frac{\partial L^2}{\partial \Psi^2}\left(1 - \frac{\partial \Psi^1}{\partial m}\right)$$

Defining $A = L^1_{w_2}/L^1_m$, $B = L^1_s/L^1_m$, $C = L^1_y/L^1_m$, $D = L^2_{w_1}/L^2_m$, $E = L^2_s/L^2_m$, and $F = L^2_y/L^2_m$. Assuming that $E \neq B$ and solving the system, we obtain the derivatives of the sharing rule

$$\Psi_{w_1} = -TC_{w_1} + \frac{BD}{E-B},$$

$$\Psi_{w_2} = \frac{AE}{E-B},$$
$$\Psi_s = \frac{BE}{E-B},$$
$$\Psi_y = \frac{CE}{E-B},$$
$$\Psi_m = \frac{E}{E-B}.$$

These partials are compatible if and only if they satisfy the usual cross-derivative restrictions. Hence, the following conditions are sufficient and necessary: (a) $\Psi_{ms} = \Psi_{sm}$, (b) $\Psi_{mw_1} = \Psi_{w_1m}$, (c) $\Psi_{mw_2} = \Psi_{w_2m}$, (d) $\Psi_{my} = \Psi_{ym}$, (e) $\Psi_{w_1w_2} = \Psi_{w_2w_1}$, (f) $\Psi_{w_1s} = \Psi_{sw_1}$, (g) $\Psi_{w_1y} = \Psi_{yw_1}$, (h) $\Psi_{w_2s} = \Psi_{sw_2}$, (i) $\Psi_{w_2y} = \Psi_{yw_2}$, (l) $\Psi_{sy} = \Psi_{ys}$, (m) $l^1_{w_1} - \frac{l^1_m}{\Psi_m}[T - l^1 + \Psi_{w_1}] \leq 0$, (m) $l^2_{w_2} - \frac{l^2_m}{1-\Psi_m}[T - l^i - \Psi_{w_2}] \leq 0$, and (o) $\frac{E}{E-B} = \frac{F+TC_y}{F-C}$. If these equations are fulfilled, then the sharing rule $\Psi(w_1, w_2, y, m, s; Z_p, Z_h)$ can be identified up to an additive constant.

3.2 The identification of the individual total cost

In this section, we show the identification result concerning the individual contribution to the total cost of production. As previously, the sharing rule $\Phi^1 = \Phi(w_1, w_2, m, s; Z_p)$ defines how household non labor income is shared between spouses. Indeed, the sharing rule $\Psi^1 = \Psi(w_1, w_2, m, y, s; Z_p, Z_h)$ defines how household non labor income minus total cost of production is shared between spouses. In the case the public good is not yet produced, namely if $y = 0$, the two sharing rule are the same object, that is:

$$\Psi(w_1, w_2, m, 0, s; Z_p, Z_h) = \Phi(w_1, w_2, m, s; Z_p).$$

Given that the two sharing rules has to be continued in $y = 0$, the previous espression always holds. This allows us to write

$$\Psi(w_1, w_2, m, y, s; Z_p, Z_h) = \Phi(w_1, w_2, m, s; Z_p) + \rho(w_1, w_2, m, y, s; Z_p, Z_h)$$

where

$$\rho(w_1, w_2, m, y, s; Z_p, Z_h) = -TC^i(w_1, w_2, m, y, s; Z_p, Z_h)$$

and

$$\rho(w_1, w_2, m, 0, s; Z_p, Z_h) = 0.$$

Let us to give you a numerical example. The sharing rule defined over the non labor income income is

$$\Phi_1(w_1, w_2, m, s) = \eta_0 + \eta_1 w_1 + \eta_2 w_2 + \eta_3 m + \eta_4 s,$$

while the sharing rule defined over the non labor income minus total production cost is

$\Psi_1 = \Psi(w_1, w_2, y, m, s) = \kappa_0 + \kappa_1 w_1 + \kappa_2 w_2 + \kappa_3 m + \kappa_4 s - \kappa_5 w_1 y - \kappa_6 w_2 y - \kappa_7 m y - \kappa_8 s y - \kappa_9 y - \kappa_{10} y^2$.

As we previously said, if public good is not yet produced, the two sharing rules are, for the household, the same object, in other words

$$\eta_0 + \eta_1 w_1 + \eta_2 w_2 + \eta_3 m + \eta_4 s = \kappa_0 + \kappa_1 w_1 + \kappa_2 w_2 + \kappa_3 m + \kappa_4 s,$$

and thus: $\eta_0 = \kappa_0$, $\eta_1 = \kappa_1$, $\eta_2 = \kappa_2$, $\eta_3 = \kappa_3$, $\eta_4 = \kappa_4$. Finally, given that the sharing rule has to be continue in $y = 0$, and that these equalities always holds, we can identify TC^1 as follows:

$$TC^1 = \kappa_5 w_1 y + \kappa_6 w_2 y + \kappa_7 m y + \kappa_8 s y + \kappa_9 y + \kappa_{10} y^2$$

where, obviously, $TC^1 = 0$ if $y = 0$.

4 The Empirical Specification

4.1 Total Cost, Labor Supplies and Sharing Rule

A. Total Production Cost. The total production cost is given by the remuneration of spouses' domestic labor supplies, t^1 and t^2, and by a market cost c^y. The market cost is equal is given by the sum of the market work bougth by the family for childcare and by a monetary cost for children goods like insurance, education, meals, transportation, etc. Then, we suppose that the total production cost has the following functional form:

$$TC(w_1, w_2, y) = A_1 w_1 y + A_2 w_2 y + A_3 \frac{w_1^2 y}{2} + A_4 \frac{w_2^2 y}{2} + A_5 w_1 w_2 y + A_6 y + A_7 y^2 + \epsilon y$$

By applying Shepard's lemma, we obtain spouses' domestic labor supplies and the demand of childcare and market goods for children.

i. Domestic labor supplies. The spouses' domestic labor supplies have the following linear form:

$$t^1(w_1, w_2, y) = A_1 y + A_3 w_1 y + A_5 w_2 y + \epsilon_1 y$$

$$t^2(w_1, w_2, y) = A_2 y + A_4 w_2 y + A_5 w_1 y + \epsilon_2 y.$$

These equations satisfy a symmetry property.

ii. Demand for childcare and market goods. The demand for childcare and market goods is obtaine as difference beteen the total cost of production and the remuneration of parents' time devoted to children.

$$c^y(w_1, w_2, y) = -\frac{A_3}{2} w_1^2 y - \frac{A_4}{2} w_2^2 y - A_5 w_1 w_2 + A_6 y + A_7 y^2 + \epsilon_3 y$$

Note that: (a) if the public good is not yet produced, namely if $y = 0$, then spouses' domestic labor suppies, the market cost for children, and, then, total production cost will be equal to zero; (b) the heteroskedastic error term in total cost function is $\epsilon = \epsilon_1 w_1 + \epsilon_2 w_2 + \epsilon_3$.

B. Sharing Rule. The next step consists in specifying the functional form for the sharing rule. The total cost function is a second-order polynomious. Thus, for symmetrical reasons, the sharing rule must be a polynomious of the same order. The husband's share has the following form:

$$\Psi_1(w_1, w_2, m, y, s) = \kappa_0 + \kappa_1 w_1 + \kappa_2 w_2 + \kappa_3 w_1^2 + \kappa_4 w_2^2 + \kappa_5 w_1 w_2 + \kappa_6 m + \kappa_7 s - \kappa_8 w_1 y - \kappa_9 w_2 y - \kappa_{10} \frac{w_1^2 y}{2} - \kappa_{11} \frac{w_2^2 y}{2} - \kappa_{12} w_1 w_2 y - \kappa_{13} my - \kappa_{14} sy - \kappa_{15} y - \kappa_{16} y^2.$$

The wife's share has the following form:

$$\Psi_2(w_1, w_2, m, y, s) = m - TC(w_1, w_2, y) - \Psi_1(w_1, w_2, m, y, s).$$

C. Total Labor Supply. The husband's total labor supply, given by the sum of market and domestic labor supplies, has the following linear structural form:

$$L^1(w_1, \Psi^1) = \alpha_0 + \alpha_1 w_1 + \alpha_2 \Psi_1 + \epsilon_4$$

with $\alpha_1 > 0$ and $\alpha_2 < 0$. Similarly for the wife:

$$L^2(w_2, \Psi^2) = \beta_0 + \beta_1 w_2 + \beta_2 \Psi_2 + \epsilon_5.$$

with $\beta_1 > 0$ and $\beta_2 < 0$.

D. Market Labor Supply. The husband's market labor supply is obtained as difference between total and domestic labor supplies:

$$h^1 = \alpha_0 + \alpha_1 w_1 + \alpha_2 \Psi_1 - A_1 y - A_3 w_1 y - A_5 w_2 y - \epsilon_2 y + \epsilon_4.$$

Similarly for the wife:

$$h^2 = \beta_0 + \beta_1 w_2 + \beta_2 \Psi_2 - A_2 y - A_4 w_2 y - A_5 w_1 y - \epsilon_1 y + \epsilon_5.$$

5 Data

The data used in this work are the French Time-Use survey (Enquête Emplois du temps) conducted by INSEE in 1998-99. The survey was designed to provide estimates of time that Frenchs spend in various activities. The survey includes a base of 8,186 households, of which 7,460 are complete (i.e. in which all household members filled in a time use booklet and an individual questionnaire). We selected a subsample of married or cohabiting couples couples with children under 18 years old. We also restrict the sample to couples in which the male reports a paid activity. This selection leave us a sample of 649 households.

The data reveal that women participation rate in employment is about 66 %. Mean monthly market working hours for men are about 158, whereas they are 129 for women. Market wages are determined as ratio between weekly labor income and hours worked per week. Given their endogeneity, we instrument wages by the level of education (from 0 -no diploma- to 8 -'Grandes Ecoles') and its second-order polynomial, the age and its second-order polynomial, the self-employment status, the country of origin of the worker (born in France or not), and the number of children by selected age group (0-2 years of age, 3-6 years of age, 7-12 years of age, and 13-18 years of age). Other instruments are related to housing: geographical area (living in Paris or not), the type of housing unit (house or flat) and the housing tenure (owned or not). Potential wage was imputed to non working women. The male hourly wage is about 10 $ per hour, whereas the wage rate of females is about 9 $ per hour.

In the theoretical model, we suppose that each spouse spends her time enjoying leisure, working in the job market and working inside the domestic walls. Domestic work is then defined as the time devoted to take care for and help household children. This definition include physical care, reading to/with children, playing with them, and all activities related to household children's education such as homework, school conferences, transportation, etc. Our data reveal that mean monthly working hours for male are about 34, whereas they are 68 for females. We also observe that parental caring type inputs are very high when children are first born and diminish as children grow up.

As we said in section 2.1, in order to produce the public good, the household buys in the market some input goods - such as clothing, school insurance, school meal, transport, school fees, etc. - and time inputs - such as a nurse, a baby-sitter, etc. Then, we defined the household monetary cost of production of the public good as c^y. The French Time-Use survey does not provide such as information then we imputed this monetary cost from the French Family Budget Survey (Enquête Budget de Familles) conducted in 2000. The variables used are the level of education and the age of both parents, the employment status of the mother (employed or not employed), the number of children by selected age group (0-2 years of age, 3-6 years of age, 7-12 years of age, and 13-18 years of age), the household total income, geographical area (regional dummy variables), and the type of childcare away from home. Our data reveal that monetary cost is decreasing in children' age, this because the purchased childcare represents a greater share of the total costs. Moreover, children monetary cost in increasing in parents' wages, other things constant. Finally, as regards the total production cost. It is decreasing in children's age till age 12 and then increase. Moreover, it is increasing in parents' wages.

Household non labor income was also imputed using the French Family Budget Survey. Household non labor income was defined as the sum saving income and income from state support for families. The variables used are the level of education, the age, and the country of origin of both parents, the number of children under 18 years of age, geographical area (regional dummy variables), the housing tenure (owned or not), and the number of rooms in the household.

Finally, the sex ratio is computed at the regional level using tha data coming from the Population Survey (Recensement de la population 1999) conducted in 1999. This is the number of men of age bwtween X and $X + 4$ divided by the number of the whole population whose age belongs to that range.

Summary statistics are reported in Table 2.

5.1 The Definition of the Quantity and Quality of Household Children

We would expect that the total cost of household children varies with factors as the number and the age of household children, the household income, the scale effect in household production and, finally, the household preferences about children. Now, we let define an index that measure the quality and quantity of household children, namely y.

What are, then, the variables that define the optimal level of the public good y? In other words, what are the variables that determine the quality and quantity of household children?

Surely, household income. Richer households may increase the quality of their children allowing them to attend better school, or lessons of music, dance...and so on. Also the number of household children has an impact on the value of y. It influence directly the quantity of household children, and indirectly their quality. A greater number of children may, in fact, lower their quality. Finally, the preferences. Let us suppose to have two households with one child of 10 years of age every and the same income. But the first household is eager to spend more for his child than the other one. In this case, the joint preferences of parents determines different values of y. Instead, we suppose that preferences are stable during the time. Then when parents decide, at the first stage of program P1, the optimal level of the public good, and then the optimal quantity and quality of children, this decision does not change during the time. Then, for this reason, we can state that the value of y does not vary with children' age.

Then, we defined the following total cost function depending on the number of children (n) and their age (age), on household income (I), on an average weight of a child when the houshold has n children (Δ) (in other words, a sort of measure of the scale effect in household production), and on a residual term (μ) that explain different household preferences about quality and quantity of household children:

$$TC = \left[\sum_{i=1}^{s} f_1(age_i)\right] * f_2(I) * \Delta(i) * exp\,(\mu)$$

where

$$f_1(age_i) = exp(\gamma_1 * age_i + \gamma_2 * age_i^2 + \gamma_3 * age_i^3)$$

$$f_2(I) = exp\,(\gamma_0 + \gamma_4 ln(I))$$

$$\Delta(i) = exp\left(\sum_{i=2}^{s} \delta_i * 1(n = i)\right)$$

Then, for example, $\Delta(2) = exp\,(\delta_2)$ is the average weight of a child when the household has two children. In Table 1 we report the estimates of $\Delta(i)$. As we can see, the average weight of a child when the household has n children decreases as the number of children increases and the marginal cost of an additional children also decreases.

Finally, once estimated parameters γ and δ, we computed the value of y using this expression:

$$\ln(y) = \hat{\gamma}_0 + \hat{\gamma}_4 ln(I) + \hat{\delta}_2 * 1(n = 2) + \hat{\delta}_3 * 1(n = 3) + \hat{\delta}_4 * 1(n = 4) + \hat{\delta}_5 * 1(n = 5) + \hat{\mu}$$

Note that the value of y does not depend on children' age because of the assumption of stable household preferences over time.

Table 1.

Number of children n	Average weight of a child when the household has n children: $\Delta(n)$	Equivalence Scale $\Delta(n) * n$	Marginal Cost
1	1	1	1
2	0.684	1.368	0.368
3	0.521	1.563	0.195
4	0.400	1.600	0.037
5	0.326	1.630	0.030

6 The Estimation Method

We estimate, by maximum likelihood, a system of five structural equations (the husband and wife's market labor supplies, the husband and wife's domestic labor supply and monetary cost), considering two different regimes: the wife participate to the labor market or not. The wife's labor force participation is based on whether her desired hours of work are greater or less than zero. The set of observations i is sorted so that the wife is working in observations 1 to k and she is not working in observations $k+1$ to n. Using an obvious notation, the system can be written as:

$$
\begin{aligned}
y_1^* &= x\beta_1^{'} + \epsilon_1, \\
y_2 &= x\beta_2^{'} + \epsilon_2, \\
y_3 &= x\beta_3^{'} + \epsilon_3, \\
y_4 &= x\beta_4^{'} + \epsilon_4,
\end{aligned}
$$

$y_5 = x\beta_5' + \epsilon_5,$

The variable y_1^* is latent; and the observables are given by:

$y_1 = y_1^*$ if $y_1^* > 0$
$y_1 = 0$ otherwise.

The system can be written more compactly as: $\mathbf{y}^i = \mathbf{x}^i\beta' + \epsilon^i$. Finally, Ω denotes the covariance matrix of ϵ.

Let us consider the two different regimes.

A. The wife participates in the labor market. If the latent representation of the wife's labor supply is greater than zero, that is,

$$y_1^* = x\beta_1' + \epsilon_1 > 0,$$

then, the observed variables are equal to the latent variables. The contribution to the likelihood function for each observation $i = 1, \ldots, k$ such that the wife participates in the labor market is the following:

$$\mathcal{L}_1^i = (2\pi)^{-5} |\Omega|^{-1/2} \exp\left[\frac{1}{2}\left(\mathbf{y}_i - \mathbf{x}_i\beta'\right)' \Omega^{-1} \left(\mathbf{y}_i - \mathbf{x}_i\beta'\right)\right]$$

B. The wife does not participate in the labor market. If the latent representation of the wife's labor supply is not greater than zero, that is,

$$y_1^* = x\beta_1' + \epsilon_1 \leq 0,$$

the wife's observed labor supply is equal to zero. The contribution to the likelihood for each observation such that the wife does not participate in the labor market is the following:

$$\mathcal{L}_2^i = (2\pi)^{-4} |\tilde{\Omega}|^{-1/2} \exp\left[-\frac{1}{2}\left(\tilde{\mathbf{y}} - \mathbf{x}\tilde{\beta}\right)' \tilde{\Omega}^{-1} \left(\tilde{\mathbf{y}} - \mathbf{x}\tilde{\beta}\right)\right] \times \Phi\left(-\frac{x\beta_1'}{\omega_{11}}\right)$$

where $\tilde{\mathbf{y}} = (y_2, y_3, y_4, y_5)'$, $\tilde{\beta} = (\beta_2, \beta_3, \beta_4, \beta_5)'$, and $\tilde{\Omega}$ is the covariance matrix of $(\epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5)$.

7 Estimation Results

The parameter estimates of the total cost function, the total labor supplies, and the sharing rule are reported, respectively, in Table 2, in Table 3, and in Table 4. The estimations show that an increase in wages and public good have a positive effect on total production cost. In particular, they reveals that an increase in man's wage reduces his domestic labor supply. Similarly for the wife because an increase in woman's wage cuts her domestic labor supply. Moreover, the negative sign of parameter A5 shows that husband and wife are substitute in domestic activities. An increase in public good has a positive effect both on husband and wife domestic labor supply. Finally, as regards the monetary cost, this is increasing with wages and the public good. Table 3 shows that an income increase has a negative effect on spouses' total labor supply. Moreover, an increase in woman's wage increases her total labor supply. Instead, for the husband, the constraint that the parameter associated to wage effect has to be greater or equal to zero is binding. Once estimated the parameters of the sharing rule, we constructed the individual total costs. Our estimates shows that for 35 households the husband's total cost is negative, while the wife's total cost is negative for 4 households. The rest of our analysis is then conducted on a sample of 610 households (instead of on the initial sample of 649 households). Let us, now, to analyse the impact of environmental variables on household ressources sharing. How does a change in husband's wage, or wife's wage, or non labor income, or sex ratio affect the way spouses share household non labor income? These results are reported in Table 5. What we can observe is that an increase of non labor income has a positive effect on the sharing rule Φ^1 (also husband's wage has a positive effect on the sharing rule but it is not significantly different from zero). On the contrary, the sharing rule is decreasing in wife's wage and in sex ratio. These results changes if we concentrate our analysis on the way spouses share household non labor income minus total production cost (see from Table 6 until Table 13). In this case, an increase in non labor income has a negative effect on the sharing rule Ψ^1 because husband's total cost TC_1 increases. In fact, what we observe is an increase in husband's monetary cost c^{y1}. On the contrary, for the wife we observe that a non labor income increase cuts her contribution to the monetary cost c^{y2}, and consequently her total cost TC^2. The last effect is a increase in her monetary ressources that she can spend for market consumption and leisure (otherwise an increase in Ψ^2). We can explain these effects saying that there is a sort of compensation by the husband with respect to his wife because the latter undergoes a greater share of the monetary cost and is more productive in household production, as Table 1 reveals. The sex ratio has not a significant effect. Let us now to analyse what happens if husband's wage increases. We observe a decrease in his contribution to the monetary cost c^{y1} and an increase in the remuneration of his domestic labor supply $w_1 t^1$. But, the final effect on his total cost TC^1, and consequently on the sharing rule Ψ^1, is not significant. For the woman we observe and increases in her contribution to the monetary cost c^{y2} and a decrease in the remuneration of her domestic labor supply $w_2 t^2$. The last effect is an increase in her total cost TC^2, and, then, a contraction of Ψ^2. What happens, instead, if wife's wage increases? What we observe is that an increase in wife's wage has not a significant effect on husband's behaviour. On the contrary, it increases the cost of her domestic labor supply $w_2 t^2$. The final effect is an increase in her total cost TC^2, and, then, a reduction of her ressources, described by Ψ^2. To conclude, we analyse the impact of a change in public good. What we observe is that, for both husband and wife, the individual total cost TC^1 and TC^2 increases because their costs of domestic labor supply increase. Neverless, this has not a significant impact on the way parents share the monetary cost.

8 Conclusion

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Table 1: Summary statistics (649 households)

	Man		Woman	
Variable	**Mean**	**Std. Dev.**	**Mean**	**Std. Dev.**
Market labor supply*	158.232	34.146	128.597	35.962
Domestic labor supply*	33.659	29.584	68.498	49.211
Hourly wage	10.294	3.963	8.640	3.078
Age	36.966	6.474	34.459	5.892
Education	3.918	2.184	4.051	2.117
Born in France (dummy)	0.960	0.196	0.955	0.207
Monetary cost**	330.607		395.637	
Total production cost**	1270.867		824.424	
Household exogenous income**	320.085		499.936	
Sex ratio	49.662		0.706	
Quality and quantity of household children	642.53		445.223	
	Man		Woman	
Estimates†	**Mean**	**Std. Dev.**	**Mean**	**Std. Dev.**
Individual total cost**	307.310	234.547	939.955	777.045
Individual monetary cost**	-35.558	353.448	360.030	527.027

*** Monthly hours worked. ** Monthly value in Euro.**

† Estimates on a sample of 610 households for which individual total cost TC^i are positive.

Table 2: Parameter estimates of the total cost function

Variable	Coefficient	(Std. Err.)
A_1	0.078***	(0.004)
A_2	0.165***	(0.006)
A_3	-0.001	(0.001)
A_4	-0.003***	(0.001)
A_5	-0.001**	(0.0005)
A_6	0.532***	(0.032)
A_7	-0.0004***	(0.00004)

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 3: Parameter estimates of total labor supply functions

Variable	Coefficient	(Std. Err.)
Man's total labor supply		
Man's wage	0	Active LB constraint
Sharing rule	-0.119***	(0.053)
Woman's total labor supply		
Woman's wage	3.449**	(1.328)
Sharing rule	-0.011***	(0.006)

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 4: Parameter estimates of the sharing rule Ψ_1

Variable	Coefficient	(Std. Err.)
κ_1	-15.561***	(3.241)
κ_2	-53.661***	(2.228)
κ_3	1.331	(1.469)
κ_4	0.451	(2.283)
κ_5	0.654	(3.520)
κ_6	0.155**	(0.082)
κ_7	-18.197*	(11.011)
κ_8	-0.201**	(0.094)
κ_9	-0.219**	(0.094)
κ_{10}	0.015*	(0.009)
κ_{11}	0.013	(0.012)
κ_{12}	0.001	(0.008)
κ_{13}	0.001**	(0.0003)
κ_{14}	0.011	(0.045)
κ_{15}	2.213	(2.490)
κ_{16}	0.00002	(0.0001)

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 5: Derivatives of the sharing rule Φ^1

Variable	Coefficient	(Std. Err.)
Man's wage	16.954	(16.096)
Woman's wage	-39.457**	(22.567)
Non labor income	0.155*	(0.082)
Sex ratio	-18.197*	(11.011)

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 6: Derivatives of the sharing rule Ψ^1

Variable	Coefficient	(Std. Err.)
Man's wage	32.993	(26.396)
Woman's wage	13.568	(22.791)
Non labor income	-0.273*	(0.139)
Public good	-0.623*	(0.342)
Sex ratio	-25.189	(29.810)

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 7: Derivatives of the sharing rule Ψ^2

Variable	Coefficient	(Std. Err.)
Man's wage	-66.387***	(26.428)
Woman's wage	-82.937***	(22.885)
Non labor income	1.273***	(0.139)
Public good	-1.337***	(0.344)
Sex ratio	25.189	(29.810)

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 8: Derivatives of the husband's total cost TC^1

Variable	Coefficient	(Std. Err.)
Man's wage	-16.040	(27.557)
Woman's wage	-53.025	(36.473)
Non labor income	0.427**	(0.214)
Public good	0.622*	(0.343)
Sex ratio	6.991	(27.421)

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 9: Derivatives of the wife's total cost TC^2

Variable	Coefficient	(Std. Err.)
Man's wage	49.433*	(27.586)
Woman's wage	122.394***	(36.528)
Non labor income	-0.427**	(0.214)
Public good	1.337***	(0.344)
Sex ratio	-6.992	(27.421)

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 10: Derivatives of the husband's monetary cost c^{y1}

Variable	Coefficient	(Std. Err.)
Man's wage	-48.717*	(27.701)
Woman's wage	-44.644	(36.579)
Non labor income	0.427**	(0.214)
Public good	0.013	(0.346)
Sex ratio	6.991	(27.421)

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 11: Derivatives of the wife's monetary cost c^{y2}

Variable	Coefficient	(Std. Err.)
Man's wage	56.248**	(27.682)
Woman's wage	58.701	(36.658)
Non labor income	-0.427**	(0.214)
Public good	0.267	(0.343)
Sex ratio	-6.992	(27.421)

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 12: Derivatives of the cost of husband's domestic work $w_1 t^1$

Variable	**Coefficient**	**(Std. Err.)**
Man's wage	29.150***	(2.476)
Woman's wage	-8.381***	(2.548)
Public good	0.584***	(0.021)

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 13: Derivatives of the cost of wife's domestic work $w_2 t^2$

Variable	**Coefficient**	**(Std. Err.)**
Man's wage	-6.814***	(2.072)
Woman's wage	54.590***	(3.677)
Public good	1.035***	(0.027)

Significance levels : $*$: 10% $**$: 5% $***$: 1%