# The tragedy of cooperation within couples Decision making and mobility

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COMMENTS WELCOME

**Abstract**

Indivividual in couple tend to behave in a cooperative manner. Empircial estimation suggest that couiple decision are Pareto. However, the litterature examine the couples in isolation, without studying the consequences of couple decision on the society. We propose the first theoretical model to study mobility decision within the couples, which involve a trade-off between couple benefits and sociatal benefit. We consider a simple mobility model and compare individual decision, couple involved in bargaining and cooperative couples.

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## 1 Introduction

Men and women spend a considerable amount of time each day to go from their home to their office. Commuting represents one of the most important activity in everyday life. There has been considerable research in this area, since it is of course important for the economy, that workers have access to their workplace under fair conditions.

The first author who has proposed a formal model to describe commuting behavior is William Vickery (see:[18]). The model works as follows: consider

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one Origin-Destination pair (Residential place and work place), one route, and one bottleneck (which can be a bridge, a tunnel, or any "weak" segment of the road, which becomes congested when the flow of vehicles is too large). A continuum of identical residents wish to arrive at destination at some ideal time $t^*$. Because the road has a finite capacity, residents will arrive earlier of later at destination, and will incur some congestion cost. Users wish to minimize the sum of congestion cost and schedule delay cost (corresponding to early or late arrival at destination). A Nash equilibrium can be computed, when no user can change his/her departure time and strictly decrease his cost.

A stochastic version of this model, when demand is described by a continuous discrete choice model was introduced in transportation by de Palma et al. [4]. Later on, Richard Arnott pointed out to the first author of this paper, that such model existed in the economics literature, and with Robin Lindsey, they extended the original Vickery model [1]. The three authors have explored the economics of the bottleneck in various dimensions incorporating various sources of heterogeneity at the demand side and the supply side (including stochastic capacity and demand). A growing literature has emerged since then, in transportation, urban economics, regional science and engineering. This research has given the momentum to the development of dynamic models, i.e. models in which the key decision is the time of use. Even if private transportation is the easiest and most obvious application, several other applications have been studied in the context of VADL (Vickrey, Arnott, de Palma & Lindsey) models. See for example (de Palma and Arnott, 1990 [16] and Forgerau and de Palma, 2009 [7]).

This literature has given an impetus to operational models which can study commuting, and non commuting behavior on large networks (the closed implementation of Vickrey model on large netorks in provided in METROPOLIS, a large-scale simulation model [8]).

We wish to propose here another extension of the seminal Vickrey model, by considering the individuals are not isolated, but part of a household, and that within this household, they share common constraints and induce externalities to each other. In other words, we wish to analize commuting models at the family level whereas, to the best of our knowledge, it is usually only studied at the individual level.

Such constraints have been sometimes taken into account either in numerical models (but with somewhat ad-hoc behaviors), or in operations research models (for example, when two traveller salesmen have to meet). But in such case, the modelling of behavior when the decision process is a partially joined process, has not been studied so far. To the best of our knowledge, this is the fist paper who introduced the idea of collective models in transportation ([5]). We believe that it is important to model the dual game of the drivers, who on one hand try to minimize his own generalized cost (sum of travel time and schedule delay cost), and on the other hand, take, at some level, into account the generalized cost (or benefit), or the other member of the family.

Since there is congestion in those system, one should be careful to evaluate the consequence of as simple behavior as cooperation, which are beneficial at the household level. However, it is less clear to know what are the social consequences (at the system level) of this cooperation. This question justifies the tittle of the current paper.

Note that we will introduce here a new version of the standard dynamic models. We will assume that the user maximizes benefit (and do not minimizes cost). This framework is different from the previous ones, and seems to us more appropriate, since it presents the commuting episode, as one episode in the 24 hours or the individual. In other works, such model would then be directly extended when other activities (e.g. shopping, leisure, etc.) will be taken into account. Such model provides a template to construct a 24 hours allocation of time model, à la G. Becker (see,[3] ).

# 2 One-population case

We first consider the one-population case. This is essentially the same model as the one proposed by Vickrey. We model the beginning of the day, starting at time zero (when everybody is at home) and finishing when the individuals arrive in their office.

Below, we introduce notations. Note, however, that the model introduced here differs from the original model, since it assumes that the individual derives a utility (benefit) during each period of time. Of course, this benefit depends on the type of activity. We consider one origin, one destination, a continuum of $N$ identical users, a single road, with capacity $s$ at the bottleneck.

## 2.1 The unit cost parameters

The unit cost parameters are:

- $\upsilon^I$: benefit from being home (alone), one minute
- $\upsilon$: benefit from being in the car, one minute
- $\upsilon^E$: benefit from being in the office (early), one minute
- $\gamma$: cost from being in the office, one minute late

We expect that:

$$\upsilon < \upsilon^E \text{ and } \upsilon < \upsilon^I$$

We also expect that :

$$\upsilon^E < \upsilon^I < \gamma.$$

## 2.2 Arrival and departure times

The Arrival and departure times are defined as follows:

- $t^d$: departure time from home

- $t^a$: arrival time in the office
- $t_q$: first departure
- $t_{q'}$: last departure
- $tt\left(t^d\right) = t^a - t^d$: travel time
- $t^*$: desired arrival time in the office (at destination).
- $B\left(.\right)$ is utility or benefits

## 2.3 Early arrive: solution

For early arrivals, the benefit function is:

$$B\left(t^d\right) = \upsilon^I t^d + \upsilon\left(t^a - t^d\right) + \upsilon^E\left(t^* - t^a\right).$$

Using the standard bottleneck model, one gets

$$\begin{array}{c} tt\left(t^d\right) = \frac{1}{s}\left[\int_{t_q}^{t} r\left(u\right)du - s\left(t - t_q\right)\right] \\ \frac{dtt\left(t^d\right)}{dt^d} = \frac{r\left(t^d\right)-s}{s} \\ \frac{d\left(t^d+tt\left(t^d\right)\right)}{dt^d} = \frac{dt^a}{dt^d} = \frac{r\left(t^d\right)}{s}. \end{array}$$

Thus, the benefit function is, after substitution:

$$B\left(t^d\right) = \left(\upsilon^I - \upsilon\right)t^d + \left(\upsilon - \upsilon^E\right)\left(t^d + tt\left(t^d\right)\right) + \upsilon^E t^*$$

The first-order condition is, with respect to the decision variable $t^d$:

$$\frac{dB\left(t^d\right)}{dt^d} = \left(\upsilon^I - \upsilon\right) + \left(\upsilon - \upsilon^E\right)\frac{r^E\left(t^d\right)}{s} = 0$$

or:

$$r^E\left(t^d\right) = \left(\frac{\upsilon^I - \upsilon}{\upsilon^E - \upsilon}\right)s.$$

Note that $\frac{\upsilon^I-\upsilon}{\upsilon^E-\upsilon} > 1$, since we assume that:

**Condition 1** $\upsilon < \upsilon^E < \upsilon^I$,

This condition means that the user prefers to stay at home alone than to stay in the office early (i.e. before the job starts). S/He enjoys less pleasure being in the car.

## 2.4 Late arrivals : solution

The benefit function for late arrivals is:

$$B\left(t^d\right) = \upsilon^I t^d + \upsilon\left(t^a - t^d\right) - \gamma\left(t^a - t^*\right).$$

After substitution we have:

$$B\left(t^d\right) = \left(\upsilon^I - \upsilon\right) t^d + \left(\upsilon - \gamma\right) t^a + \gamma t^*.$$

and the first-order conditions are:

$$\frac{dB\left(t^d\right)}{dt^d} = \left(\upsilon^I - \upsilon\right) + \left(\upsilon - \gamma\right)\frac{r\left(t^d\right)}{s} = 0$$

Thus the equilibrium solution is:

$$r^L\left(t^d\right) = \left(\frac{\upsilon^I - \upsilon}{\gamma - \upsilon}\right) s.$$

Note that $0 < \frac{\upsilon^I - \upsilon}{\gamma - \upsilon} < 1$, since we assume:

**Condition 2** $\upsilon < \upsilon^I < \gamma$.

These conditions imply that being in the car is less enjoyable than being home, and that the benefit of being at home (one unit of time) is smaller than the cost of being one unit of time late (losses and more severe than gains, as prospect theory teaches us, see, e.g. [17]).

Altogether, we have:

**Condition 3 (Ranking Unit Benefit)** $\upsilon < \upsilon^E < \upsilon^I < \gamma$.

Note that, without loss of generality, we can assume that: $\upsilon = 0$.

### 2.4.1 Individual benefit

At equilibrium, everybody has the same individual benefit. The level of individual benefit is the same for all users, but the composition of this benefit differs, from one user to the next user. The social cost can be derived as follows:

$$B\left(t_q\right) = \left(\upsilon^I - \upsilon^E\right) t_q + \nu^E t^* = \left(\upsilon^I - \gamma\right) t_{q'} + \gamma t^*$$
$$\left(t_{q'} - t_q\right) s = N.$$

This system of tow equations and two unknown implies that:

$$\left(\gamma - \upsilon^E\right) t_q + \upsilon^E t^* = \left(\upsilon^I - \gamma\right)\frac{N}{s} + \gamma t^*$$

The time of the first departure and of the last departure are then equal to:

$$\begin{aligned} t_q &= t^* - \frac{N}{s}\frac{\left(\gamma - \upsilon^I\right)}{\left(\gamma - \upsilon^E\right)} \\ t_{q'} &= t^* + \frac{N}{s}\frac{\left(\upsilon^I - \upsilon^E\right)}{\left(\gamma - \upsilon^E\right)}. \end{aligned}$$

Therefore:

**Proposition 4** *The equilibrium time benefit of the extended VADL (Vickery, Arnott, dePalma & Lindsey) model is given by:*

$$(B^*)_{\text{Single}} = \upsilon^I t^* - \frac{N}{s}\frac{\left(\gamma - \upsilon^I\right)\left(\upsilon^I - \upsilon^E\right)}{\left(\gamma - \upsilon^E\right)}.$$

If transportation were instantaneous (teleportation), capacity would be infinite, all individuals would arrive on time, and the benefit would be $\upsilon^I t^*$. Because the capacity is limited, the benefit is smaller by a cost term $C^*$, with

$$C^* = \frac{N}{s}\frac{\left(\gamma - \upsilon^I\right)\left(\upsilon^I - \upsilon^E\right)}{\left(\gamma - \upsilon^E\right)}.$$

The term represents the opportunity cost of not being at home (which maximized the benefit, per unit of time).

Note that $C^*$ is minimized for $\upsilon^I = \upsilon^E$, and maximized for $\upsilon^I = \frac{\gamma + \upsilon^E}{2}$, so that:

$$0 < C^* < \frac{N}{s}\frac{\left(\gamma - \upsilon^E\right)}{4}.$$

Condition 3 implies that $C^* > 0$.

The result should be compared with the standard results. In this case, the equilibrium cost, denoted by $C^{\text{Wickrey}}$ is given by

$$C^{\text{Wickrey}} = \frac{\upsilon^E \gamma}{\upsilon^E + \gamma}.$$

# 3 Couple - Nash solution, two separate routes

We assume in this section that men and women are selfish, and they all maximize their own benefit, regardless of the benefit of the other spouse.

## 3.1 The commuting problem for the couples

A subscript, $m$, $w$, and $c$, refer to: "man", woman", and "couple".

**Definition 5** *Singles get married if their valuation of time at home together is larger than their valuation of time at home being alone.*

[to be discussed...]

We consider a situation where men and women live in couple. They both work, and each has a car. They use different route. As a matter of fact, all men work in the factory located at $D_m$ and all women work in the offices, located at $D_w$. The route going from $O$ (the residential place), to $D_m$ has a capacity denoted by $s_m$. Likewise, the route going from $O$ to $D_w$ has capacity denoted by $s_w$. We assume that without congestion, the man should leave earlier than the woman, in order to arrive on time.

## 3.2 Early arrivals for man and woman - man leaves before woman

We assume that men leave earlier than women, so that the benefit for the men and the women are not symmetric. The condition that the man leaves before the woman can be understood (for the time being) as follows: the woman has the single key of the house and therefore leaves after her spouse.

- $\upsilon_m^C$: benefit from being home in couple one minute, for the men

### 3.2.1 For the man (early):

$$\begin{array}{c} B_m\left(t_m^d\right) = \upsilon_m^c t_m^d + \upsilon_m\left(t_m^a - t_m^d\right) + \upsilon_m^E\left(t_m^* - t_m^a\right) \\ B_m\left(t_m^d\right) = \left(\upsilon_m^c - \upsilon_m\right) t_m^d + \left(\upsilon_m - \upsilon_m^E\right) t_m^a + \upsilon_m^E t_m^* \end{array} .$$

### 3.2.2 For the woman (early):

$$\begin{array}{c} B_w\left(t_w^d\right) = \upsilon_w^c t_w^d + \nu_w^I\left(t_w^d - t_m^d\right) + \upsilon_w\left(t_w^a - t_w^d\right) + \upsilon_w^E\left(t_w^* - t_w^a\right) \\ B_w\left(t_w^d\right) = \left(\upsilon_w^c - \upsilon_w^I\right) t_m^d + \left(\upsilon_w^I - \upsilon_w\right) t_w^d + \left(\upsilon_w - \upsilon_w^E\right) t_w^a + \upsilon_w^E t_w^* \end{array}$$

### 3.2.3 Solution of a Nash equilibrium

Assume that men and women do not use the route at the same time. Then, for the men; we have:

$$\begin{aligned} \frac{dB_m\left(t_m^d\right)}{dt_m^d} &= \left(\upsilon_m^c - \upsilon_m\right) + \left(\upsilon_m - \upsilon_m^E\right)\frac{r_m^E\left(t^d\right)}{s} = 0 \\ r_m^E\left(t^d\right) &= \left(\frac{\upsilon_m^c - \upsilon_m}{\upsilon_m^E - \upsilon_m}\right) s. \end{aligned}$$

We have the same solution as for the singles, except that $\nu_m^I \to \upsilon_m^c$. Since they are married: $\upsilon_m^c > \nu_m^I$. The result can be understood as follows. The man (who leaves earlier) adjust his margin with respect of being in couple at home, rather than from being alone at home.

For the woman, we have:

$$\begin{aligned}
\frac{dB_w\left(t_w^d\right)}{dt_w^d} &= \left(\upsilon_w^I - \upsilon_w\right) t_m^d + \left(\upsilon_w - \upsilon_w^E\right) * \frac{r_w^E\left(t^d\right)}{s} = 0 \\
r_w^E\left(t^d\right) &= \left(\frac{\upsilon_m^I - \upsilon_m}{\upsilon_m^E - \upsilon_m}\right) s.
\end{aligned}$$

This is the same solution as for the single case.

Note that $r_m^E\left(t^d\right) > r_w^E\left(t^d\right)$, since $\upsilon_m^c > \upsilon_m^I$, i.e.

**Proposition 6** *At equilibrium, men encounter more congestion than women if the benefit of being together, for the women, is larger than the benefit of being alone, for the women.*

The condition $\upsilon_m^c > \upsilon_m^I$ is likely to be true empirically.

### 3.3 Late arrivals for man and woman - man leaves before woman

For late arrivals we have, for the man

$$\begin{aligned}
B_m\left(t_m^d\right) &= \upsilon_m^c t_m^d + \upsilon_m\left(t_m^a - t_m^d\right) - \gamma_m\left(t_m^a - t_m^*\right) \\
B_m\left(t_m^d\right) &= \left(\upsilon_m^c - \upsilon_m\right) t_m^d + \left(\upsilon_m - \gamma_m\right) t_m^a + \gamma_m t_m^*
\end{aligned}.$$

Therefore:

$$\frac{dB_m\left(t_m^d\right)}{dt_m^d} = \left(\upsilon_m^c - \nu_m\right) + \left(\nu_m - \gamma_m\right)\frac{r_m^L\left(t^d\right)}{s} = 0.$$

So that:

$$\left\{r_m^L\left(t^d\right)\right\}_{\text{Nash}} = \frac{\left(\upsilon_m^c - \upsilon_m\right)}{\left(\gamma_m - \nu_m\right)} s,$$

which is the same as in the single case, except that now $\upsilon_m^I \to \upsilon_m^c > \upsilon_m^I$.

And for the woman:

$$\begin{aligned}
B_w\left(t_w^d\right) &= \upsilon_w^c t_w^d + \upsilon_w^I\left(t_w^d - t_m^d\right) + \upsilon_w\left(t_w^a - t_w^d\right) - \gamma_w\left(t_w^a - t_w^*\right) \\
B_w\left(t_w^d\right) &= \left(\upsilon_w^c - \upsilon_w^I\right) t_m^d + \left(\upsilon_w^I - \upsilon_w\right) t_w^d + \left(\upsilon_w - \gamma_w\right) t_w^a + \gamma_w t_w^*.
\end{aligned}$$

Therefore:

$$\frac{dB_w\left(t_w^d\right)}{dt_w^d} = \left(\upsilon_w^I - \upsilon_w\right) + \left(\upsilon_w - \gamma_w\right)\frac{r_w^L\left(t^d\right)}{s}.$$

So that:

$$\left\{r_w^L\left(t^d\right)\right\}_{\text{Nash}} = \frac{\left(\upsilon_m^I - \upsilon_m\right)}{\left(\gamma_m - \upsilon_m\right)} s,$$

which is the same as in the single case.

### 3.4 Global condition

We have used first order condition insofar. It remains to argue that the man does not wish to live earlier than the woman, and that likewise, the woman does not wish to leave before the man.

Assume that the man leaves before the woman. Living one minute later yields a benefit of staying at home is $\Delta BH = \upsilon_m^c$, and therefore, necessarily the transport cost increase is $\Delta TC = \upsilon_m^c$. Now, if the man leaves one minute later, but leaves later than the women, by one minute, the transport cost increase is still $\Delta TC = \upsilon_m^c$, while the benefit of staying at home is $\Delta BH = \upsilon_m^I$, therefore, the total benefit from this last move is: $\Delta BH - \Delta TC = \upsilon_m^I - \upsilon_m^c < 0$. Likewise, it can be shown than the woman has no incentive to leave before the man. The condition: the woman has the single key of the house and therefore leaves after his spouse is therefore not necessary. It should be the case at equilibrium.

### 3.5 Equilibrium cost

The discussion is trivial for the woman, who leaves later. We focus here on the man, who leaves earlier. We conjecture that the man's benefit in this case is:

$$(B_m^*)_{\text{Couple}} = \upsilon_m^c t_m^* - \frac{N}{s} \frac{(\gamma_m - \upsilon_m^c)\left(\upsilon_m^c - \upsilon_m^E\right)}{(\gamma_m - \upsilon_m^E)}.$$

Note that it is likely that the social cost is not unique in this problem, since departure time of men can be swapped between two couples, leaving congestion cost constant, but decreasing the cost of women.

To keep things comparable, let us assume that spouses have the same parameter values as the single (except that they value more being in couple than alone):

$$\begin{gathered}(B_m^*)_{\text{Couple}} - (B_m^*)_{\text{Single}} = \\ \tfrac{N}{s} \tfrac{1}{(\gamma_m - \nu_m^E)} \left[\left(\gamma_m - \nu_m^I\right)\left(\upsilon_m^I - \upsilon_m^E\right) - (\gamma_m - \upsilon_m^c)\left(\upsilon_m^c - \upsilon_m^E\right)\right].\end{gathered}$$

or, after some reorganization:

$$(B_m^*)_{\text{Couple}} - (B_m^*)_{\text{Single}} = \frac{N}{s} \frac{\left(\upsilon_m^c - \upsilon_m^I\right)}{(\gamma_m - \nu_m^E)} \left[\left(\upsilon_m^c + \upsilon_m^I\right) - \left(\gamma_m + \upsilon_m^E\right)\right].$$

The differential (married versus being single) opportunity cost of travel is given by the My's factor: $\Xi$, with:

$$\Xi = \left(\upsilon_m^c + \upsilon_m^I\right) - \left(\gamma_m + \upsilon_m^E\right) = \underset{>0}{\left(\upsilon_m^I - \upsilon_m^E\right)} - \underset{>0}{(\gamma_m - \upsilon_m^c)}$$

Recall that:

$$\nu_m^E < \nu_m^I < \nu_m^c < \gamma_m.$$

So that the My's factor can be positive of negative for the man.

**Proposition 7 (Men getting married)** *The differential opportunity cost of travel for the man (leaving earlier) $\Xi$ can be negative or positive, being in couple, compared to the benefit of being single. The condition depends on the unit benefits of being alone, together, and of being early or late in the office. $\Xi$ is positive if:*

$$\left(\upsilon_m^I - \upsilon_m^E\right) > \left(\gamma_m - \upsilon_m^c\right).$$

For the woman, we have:

$$\begin{aligned}
\frac{dB_w\left(t_w^d\right)}{dt_w^d} &= \left(\upsilon_w^I - \upsilon_w\right) + \left(\upsilon_m - \upsilon_w^E\right) * \frac{r_w^E\left(t^d\right)}{s} = 0 \\
r_w^E\left(t^d\right) &= \left(\frac{\upsilon_w^I - \upsilon_w}{\upsilon_w^E - \upsilon_w}\right) s.
\end{aligned}$$

The opportunity cost for the women is the same before and after marriage, since she is facing at the margin being alone, as when she was single.

Note that this is not optimal, since the men does not take into account in his decision, the benefit of his spouse.

**Proposition 8 (Women getting married)** *The differential opportunity cost of travel for the woman (leaving late) is null, i.e. she does not changes her travel pattern.*

### 3.5.1 Getting married: the impact on congestion

Assume men and women have the same parameter values (but different values of desired arrival time). We also assume that

**Condition 9** $\upsilon^C > \upsilon^I$, *i.e. individual prefer to stay with the spouse than together.*

**Proposition 10** *When singles get married and appreciate more being together than being alone, congestion cost increases iff $\upsilon^c > \upsilon$.*

**Proof.** It suffices to note that ∎

$$\begin{aligned}
\underbrace{\frac{r^E\left(t^d\right)}{s}}_{\text{Single Early}} &= \left(\frac{\upsilon^I - \upsilon}{\upsilon^E - \upsilon}\right) < \underbrace{\frac{r_m^E\left(t^d\right)}{s}}_{\text{Nash(Couple) Early}} = \left(\frac{\upsilon^c - \upsilon}{\upsilon^E - \upsilon}\right), \\
\underbrace{\frac{r_w^L\left(t^d\right)}{s}}_{\text{Single Late}} &= \frac{\left(\upsilon_m^I - \upsilon_m\right)}{\left(\gamma_m - \upsilon_m\right)} = \underbrace{\frac{r_m^E\left(t^d\right)}{s}}_{\text{Nash(Couple) Late}}
\end{aligned}$$

The reason is that the first spouse to depart has now a larger benefit of staying at home, and therefore less incentive to leave. This creates congestion.

# 4 Couple - Nash solution, one single destination and one route

We assume here that the work locations of men and women are located in the same place. there is a single route from the origin to the destination, and it is shared by men and women. (To be done).

# 5 Cooperative solution

We assume here that the men and the women in the same household fully cooperate. In this case, they are concerned with the total benefit of the couple. There is no implications at this point on how the couple marginal benefit from cooperation (which is positive by definition), is shared.

## 5.1 Equilibrium

The benefit for the couple, is then:

$$\begin{aligned} B_c\left(t_m^d, t_w^d\right) &= \left(\upsilon_m^c - \upsilon_m\right) t_m^d + \left(\upsilon_m - \upsilon_m^E\right) t_m^a + \upsilon_m^E t_m^* + \\ & \left(\upsilon_w^c - \upsilon_w^I\right) t_m^d + \left(\upsilon_w^I - \upsilon_w\right) t_w^d + \left(\upsilon_w - \upsilon_w^E\right) t_w^a + \upsilon_w^E t_w^* \end{aligned}$$

Assume again, that the congestion for the men and the women are independent. Then, for the man, we have:

$$\begin{aligned} \frac{B_c\left(t_m^d, t_w^d\right)}{dt_m^d} &= \left(\upsilon_m^c - \upsilon_m\right) + \left(\upsilon_w^c - \nu_w^I\right) + \left(\upsilon_m - \upsilon_m^E\right) * \frac{r_m\left(t^d\right)}{s} = 0 \\ \left\{\frac{r_m^E\left(t^d\right)}{s}\right\}_{\text{Coop}} &= \frac{1}{\upsilon_m^E - \upsilon_m}\left(\upsilon_m^c + \underbrace{\left(\upsilon_w^c - \upsilon_w^I\right)}_{\text{Extra term} > 0} - \upsilon_m\right). \end{aligned}$$

Recall that for Nash, we have $\left\{r_m^E\left(t^d\right)\right\}_{\text{Nash}} = \left(\frac{\upsilon_m^c - \upsilon_m}{\upsilon_m^E - \upsilon_m}\right)$, so:

$$\left\{\frac{r_m^E\left(t^d\right)}{s}\right\}_{\text{Coop}} = \left\{r_m^E\left(t^d\right)\right\}_{\text{Nash}} + \underbrace{\frac{\upsilon_w^c - \upsilon_w^I}{\upsilon_m^E - \upsilon_m}}_{>0}.$$

This analysis suggests that the departure rate under a cooperative solution involves more congestion.

## 5.2 Computation of the equilibrium cost, in the cooperative case:

For early arrival, we have:

$$\begin{aligned} B_c\left(t_{q;m}, t_{q,w}\right) &= \left(\upsilon_m^c - \upsilon_m\right) t_{q,m} + \left(\upsilon_m - \upsilon_m^E\right) t_{q,m} + \upsilon_m^E t_m^* + \\ & \left(\upsilon_w^c - \upsilon_w^I\right) t_{q,m} + \left(\upsilon_w^I - \upsilon_w\right) t_{q,w} + \left(\upsilon_w - \upsilon_w^E\right) t_{q,w} + \upsilon_w^E t_w^* \end{aligned}$$

Or:

$$\begin{aligned} B_c\left(t_{q;m}, t_{q,w}\right) &= \left(\left(\upsilon_m^c - \upsilon_m^E\right) + \left(\upsilon_w^c - \upsilon_w^I\right)\right) t_{q,m} + \upsilon_m^E * t_m^* + \\ & \left(\upsilon_w^I - \upsilon_w^E\right) t_{q,w} + \upsilon_w^E t_w^*. \end{aligned}$$

For late arrivals, we have:

$$\begin{aligned} B_c\left(t_{q';m}, t_{q',w}\right) &= \left(\upsilon_m^c - \upsilon_m\right) t_{q',m} + \left(\upsilon_m - \gamma_m\right) t_{q',m} + \gamma_m t_m^* + \\ & \left(\upsilon_w^c - \upsilon_w^I\right) t_{q',w} + \left(\upsilon_w^I - \upsilon_w\right) t_{q',m} + \left(\upsilon_w - \upsilon_w\right) t_{q',m} + \gamma_w t_w^* \end{aligned}$$

Or:

$$\begin{aligned} B_c\left(t_{q';m}, t_{q',w}\right) &= \left(\left(\upsilon_m^c - \upsilon_m^E\right) + \left(\upsilon_w^c - \upsilon_w^I\right)\right) t_{q',m} + \gamma_m t_m^* + \\ & \left(\upsilon_w^I - \gamma_m\right) t_{q',m} + \gamma_w t_w^*. \end{aligned}$$

By subscription, we get: (needs check and completion), four equations and two unknowns,.... . $t_{q',w}, t_{q,w}, t_{q',m}, tm.$

$$\begin{gathered} B_c\left(t_{q';m}, t_{q',w}\right) - B_c\left(t_{q;m}, t_{q,w}\right) \\ = \left(\left(\upsilon_m^c - \upsilon_m^E\right) + \left(\upsilon_w^c - \upsilon_w^I\right) + \upsilon_w^I\right)\left(t_{q',m} - t_{q,m}\right) \\ -\gamma_m\left(\frac{N}{s} + t_{q,m}\right) + \nu_w^E * t_{q,w} + \left(\gamma_m - \nu_m^E\right) * \left(t_m^* + t_w^*\right) \\ = 0 \end{gathered}$$

# 6 The regimes will be determined endogenously:

Departure regimes

- D: M/W (man first, then woman)
- D: W/M (woman first, then man)
- D: T (man and women together)

Arrival regimes

- Arrival : Early-Early (early man and early women)
- Arrival: Late-Late (late man and late woman)
- Arrival Early-Late (early man and late woman)
- Arrival: Late-Early (late man and early woman)

Congestion

- Men and women use the same route
- Men use route 1 and women use route 2 (or the same route, but with no overlap).

Comparison of the two regimes at the aggregate level: cooperation, versus non-cooperation. The social cost is not uniquely determined. It depends on how the couples are twined (i.e if two men shift their departure rate, keeping the departure rate of their spouses, fixed, congestion will remain the same, but the benefit for the women (for example being less long alone at home), may increase. I suspect that there is a continuum of social costs.

# References


[1] Arnott, R., A. de Palma and R. Lindsey (1992). A Structural Model of Peak-Period Congestion: A Traffic Bottleneck with Elastic Demand, *American Economic Review*, 83, 161-179.

[2] Ashiru, O., Polak, J.W. and Noland, R.B. (2003), "Development and application of an activity based space-time accessibility measure for individual activity schedules", *Proceedings of the European Regional Science Conference*, Jyväskylä, Finland.

[3] Becker, G. S. (1965) "A Theory of the Allocation of Time," *Economic Journal* 75 (299), pp. 493-517.

[4] de Palma, A., Ben-Akiva, Cl. Lefèvre and N. Litinas (1983), Stochastic Equilibrium Model of Peak Period Traffic Congestion, *Transportation Science*, 17(4), 430-453.

[5] Chiappori, P.-A. (1998)

[6] de Palma, A., S. Proost and F. Dunkerly (2010). Trip chaining, who win, who loose? *Journal of Economics and Management Science*, forthcoming.

[7] de Palma, A. and Fosgerau, M. (2009) Random queues and risk averse users. Unpublished manuscripts, Ecole Polytechnique, France.

[8] de Palma, A., F. Marchal and Y. Nesterov (1997). "METROPOLIS: A Modular System for Dynamic Traffic Simulation", *Transportation Research Record* 1607, 178-184, Washington, D.C.

[9] Collins, M. and Z. Berge, (1994), "Professional and Technical Moonlighters", Electronic Journal of Sociology, ISSN: 1198 3655.

[10] Olu, A., J. W. Polak and R. B. Noland (2003), "Space-time user benefit and utility accessibility measures for individual activity schedules", 82nd Annual Meeting of the Transportation Research Board.

[11] Ashiru, O., J.W. Polak and R.B. Noland (2003), "Space-time user benefit and utility accessibility measures for individual activity schedules", Transportation Research Record 1854, pp. 62-73.

[12] Ashiru, O., Polak, J.W. and Noland, R.B. (2004), "The utility of schedules: A model of departure time choice and activity time allocation with application to individual activity", *Transportation Research Record* 1894, pp. 84-98.

[13] Schintler, L. (2001), "Women and travel", in D.A. Hensher and K.J. Button (eds.), Handbook of Transport Systems and Traffic Control 3, Oxford: Elsevier Science, 351-358.

[14] Sivakumar, A. and Polak, J.W. (2009), “Modelling the endogeneity in activity participation and technology holdings”,paper presented to the 1st International Conference on Choice Modelling, March 2009.

[15] Small, K. (1982) The scheduling of Consumer Activities: Work Trips. *American Economic Review* 72(3), 467-479.

[16] de Palma, A. and R. Arnott (1989/1990), The Temporal Use of a Telephone Line, *Information Economics and Policy*, 4, 155-174.

[17] Kahneman , D. and Tversky , A . ( 1979 ). Prospect theory: an analysis of decision under risk . Econometrica 47 , 263 − 292 .

[18] Vickrey, W.S. (1969). "Congestion Theory and Transport Investment". *American Economic Review* (Papers and Proceedings) 59, 251-261.